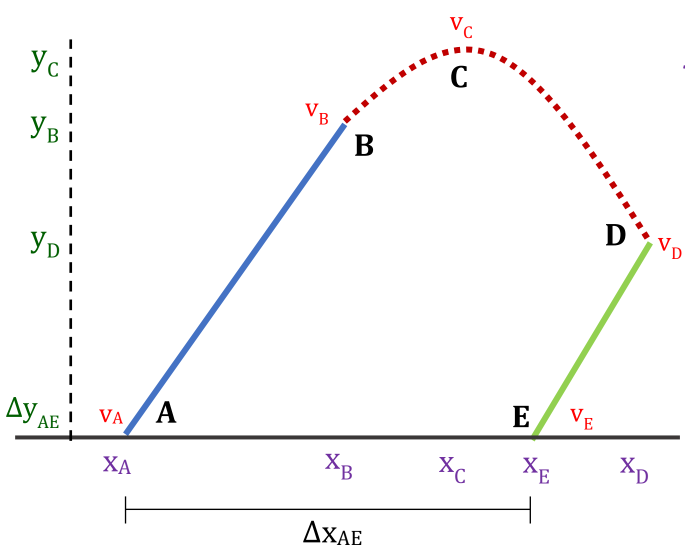
Kiara Lavana 10/07/2021 Physics Section Q

Über Rocket with Algebra Alex

Description

One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at 39˚ above the East horizontal. The rocket engine is designed to burn for 7.2 s while producing a constant net acceleration of 4.1 m/s^2 for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops, the rocket continues in projectile motion. A parachute opens after the rocket falls 62 m from its maximum height. When the parachute opens the rocket instantly changes speed and descends at 10 m/s. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind (14 m/s). Assume the wind affects the rocket only during the parachute stage.

Diagram:



Givens:

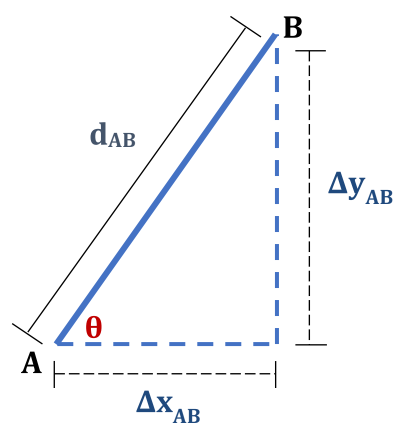
$v_A = 0$ m/s	$v_{DX} = -14$ m/s
$v_{DY} = -10$ m/s	$v_{CY} = 0$ m/s
$\theta = 39˚$	$t_{AB} = 7.2$ s
$a_{AB} = 4.1$ m/s^2	$a_{DE} = 0$ m/s^2
$\Delta y_{CD} = -62$ m	$y_E = 0$ m
$a_y = -9.8$ m/s^2	$a_x = 0$ m/s^2

Stage 1: Using the diagonal displacement from the rocket launch, determine the initial velocity and intial position at point B in order to set up a standard projectile motion question

Calculating the diagonal net displacement from A to B:

$$d_{AB} = \frac{1}{2}a_{AB}t_{AB}^2 + v_A t_{AB} + y_A$$
$$d_{AB} = \frac{1}{2}(4.1)(7.2)^2$$
$$d_{AB} = 106.272 \text{ m}$$

Calculating x and y position at B using the x and y components of d_{AB}:

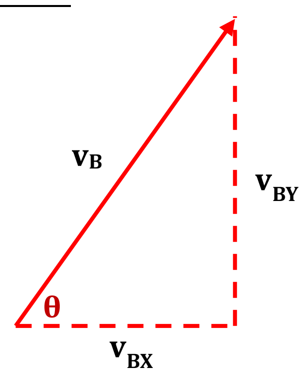


$$y_B = d_{AB}\sin\theta$$
$$y_B = 106.272 \sin 30$$

$$x_B = d_{AB}\cos\theta$$
$$x_B = 106.272 \cos 39$$

Calculating v_B and then v_{BX} and v_{BY}:

$$v_B = a_{AB}t_{AB} + v_A$$
$$v_B = (4.1)(7.2)$$
$$v_B = 29.52 \text{ m/s}$$



$v_{BX} = v_B \sin\theta$
$\underline{v_{BX} = 29.52 \sin 39}$

$v_{BY} = v_B \cos\theta$
$\underline{v_{BY} = 29.52 \cos 39}$

Stage 2: Determine the x and y position at point C

Calculating t_{BC} to find x_C:
$v_{CY} = a_y t_{BC} + v_{BY}$
$0 = (-9.8)t_{BC} + (29.52 \sin 39)$
$\underline{t_{BC} = 1.896 \text{ s}}$

$x_C = \frac{1}{2}a_X t_{BC}^2 + v_{BX} t_{BC} + x_B$
$x_C = (29.52 \cos 39)(1.896) + (106.27 \cos 39)$
$x_C = 43.4892 + (106.272 \cos 39)$
$\underline{x_C = 126.078 \text{ m}}$

Calculating y_C using v_{CY}, v_{BY}, and y_B:
$v_{CY}^2 = v_{BY}^2 + 2a_Y(y_C - y_B)$
$0 = (29.5 \sin 39)^2 + 2(-9.8)(y_C - 106.2 \sin 39)$
$0 = 345.125 - 19.6y_c + 1310.83$
$0 = -19.6y_c + 1655.96$
$-19.6y_c = 1655.96$
$\underline{y_C = 84.488 \text{ m}}$

Stage 3: Find the x and y position at point D

Find y_D from Δy_{CD} and y_C:
$\Delta y_{CD} = y_D - y_C$
$-62 = y_D - 84.488$
$\underline{y_D = 22.4878 \text{ m}}$

Calculate t_{CD} using y_C and y_D:
$y_D = \frac{1}{2}a_y t_{CD}^2 + v_{CY} t_{CD} + y_C$
$22.4876 = \frac{1}{2}(-9.8)t_{CD}^2 + 84.488$
$22.4876 = -4.9t_{CD}^2 + 84.488$
$4.9t_{CD}^2 = 62.0004$
$t_{CD}^2 = 12.6531$
$\underline{t_{CD} = 3.55713 \text{ s}}$ or ~~$t_{CD} = -3.55713$ s~~

Calculate x_D using t_{CD}, v_{CX}, and x_C:
$x_D = \frac{1}{2}a_X t_{CD}^2 + v_{CX} t_{CD} + x_C$
$x_D = (29.52 \cos 39)(3.55713) + 126.078$
$x_D = 81.6053 + 126.078$
$\underline{x_D = 207.683 \text{ m}}$

Stage 4: Determine the x position of point E to calculate x displacement from point A to D

Calculate t_{DE} using y_E, y_D, and v_{DY}
$y_E = \frac{1}{2}a_{DE} t_{DE}^2 + v_{DY} t_{DE} + y_D$
$0 = -10t_{DE} + 22.4476$
$10t_{DE} = 22.4476$
$\underline{t_{DE} = 2.2487 \text{ s}}$

Calculate x_E using v_{DX}, t_{DE}, and x_D:
$x_E = \frac{1}{2}a_X t_{DE}^2 + v_{DX} t_{DE} + x_D$
$x_E = (-14)(2.24876) + 207.683$
$x_E = -31.4826 + 207.683$
$\underline{x_E = 176.201}$

$\Delta x_{AE} = x_D - x_A$
$\boxed{\Delta x_{AE} = 176.2 \text{ m East}}$