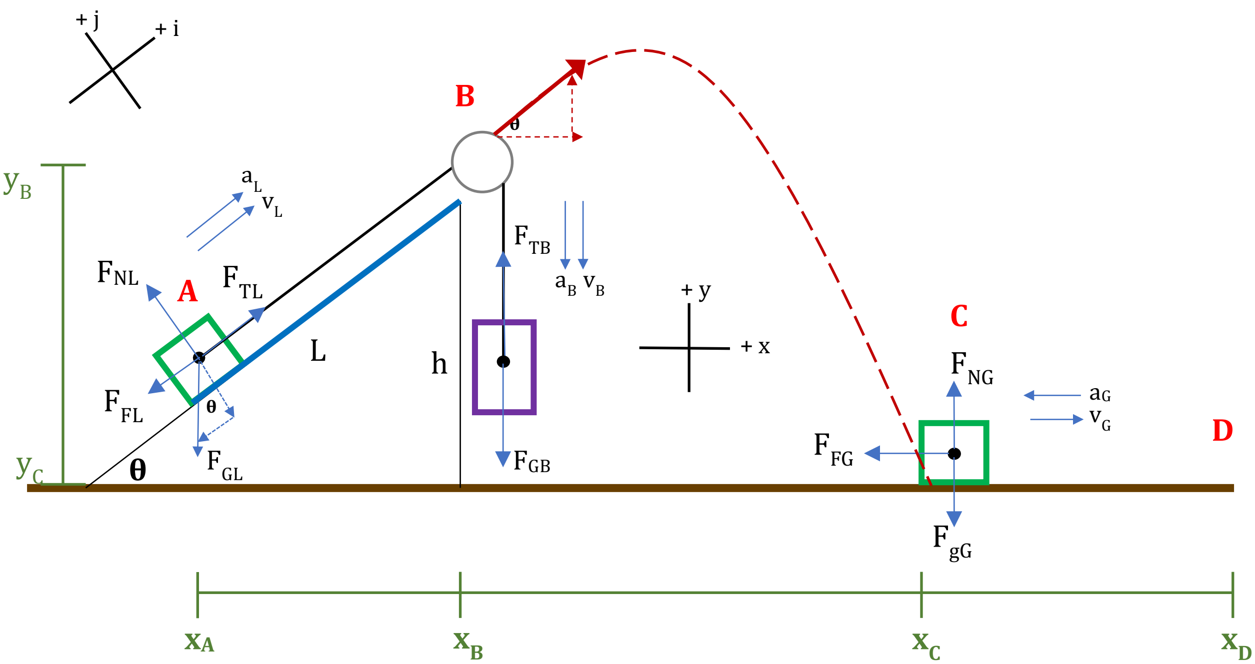
Kiara Lavana 2 November 2021 Section Q

Über Problem: Leaping Larry's Luge Launcher

Description:
Leaping Larry decided to make a laborious launcher for his luxury luge using a pulley and ramp system (see diagram). His method was to attach one end of a massless stretchless rope to a barrel of rocks and to hold the other end of the rope. He placed the rope over a massless frictionless pulley, and then walked down the ramp far as down possible to point A (where L = h). When he sat in the luge, he accelerated up the ramp to point B and then launched off the top at the same angle as the ramp (all while releasing the rope and avoiding the pulley). He flew through the air as a projectile to point C, smoothly transitioning all his (net) speed into the horizontal direction, and eventually slid to a stop at point D.

Diagram:



Givens:
$m_L = 42$ kg
$m_B = 53$ kg
$\theta = 29°$
$\mu_R = 0.25$
$h = 6.8$ m
$h = L = -y_{BC}$
$\Delta x_{BD} = 75$ m

Assumptions:
$F_{TL} = F_{TB}$
$a_L = -(a_B)$

STAGE 1: Using the forces on the luge in the pulley/ramp system, determine the acceleration of the luge on the ramp.

Write out the sum of forces in the various directions acting on the two objects.

$\sum F_{jL} : F_{NL} - F_{g1}\cos\theta = m_L a_{LY}$
$F_{NL} - m_L g\cos\theta = 0$
$\underline{F_{NL} = m_L g\cos\theta}$

$\sum F_{iL} = F_{TL} - F_{FL} - F_{gL}\sin\theta = m_L a_{Lx}$
$F_{TL} - \mu_R F_{NL} - m_L g\sin\theta = m_L a_L$
$F_{TL} - \mu_R m_L g\cos\theta - m_L g\sin\theta = m_L a_L$
$\underline{F_{TL} = m_L a_L + \mu_R m_L g\cos\theta + m_L g\cos\theta}$

$\sum F_{yB} : F_{TB} - F_{gB} = m_B a_B$
$F_{TB} - m_B g = m_B a_B$
$\underline{F_{TB} = m_B a_B + m_B g}$

Set $F_{TB} = F_{TL}$
$m_B a_B + m_B g = m_L a_L + \mu_R m_L g\cos\theta + m_L\sin\theta$
$-m_B a_L + m_B g = m_L a_L + \mu_R m_L g\cos\theta + m_L\sin\theta$
$-m_B a_L - m_L a_L = \mu_R m_L g\cos\theta + m_L g\sin\theta - m_B g$
$-a_L(m_B + m_L) = \mu_R m_L g\cos\theta + m_L g\sin\theta - m_B g$
$-a_L(95) = (0.25 \cdot 42 \cdot 9.8\cos 29) + 42 \cdot 9.8\sin 29 - 53 \cdot 9.8$
$-95a_L = 89.9984 + 199.548 - 519.4$
$\underline{a_L = 2.41952 m/s^2}$

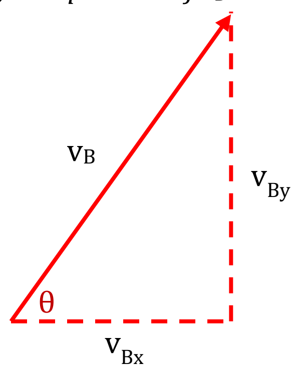
STAGE 2: Find v_B (the initial velocity of the projectile motion portion) using the calculated acceleration in order to set up a standard projectile motion question.

Use a_L, L, and v_A = 0 m/s to determine v_B with the 4th kinematics equation.

$v_B^2 = v_A^2 + 2a_L L$
$v_B^2 = 2(2.41952)(6.8)$
$v_B^2 = 32.9054$
$\underline{v_B = 5.73631\ m/s}$

STAGE 3: Find the net v_C after calculating its x and y components using the kinematics equations with projectile motion.

Find the x and y components of v_B.



$v_{Bx} = v_B \sin\theta$
$\underline{v_{Bx} = 5.73631 \sin 29}$

$v_{By} = v_B \cos\theta$
$\underline{v_{By} = 5.73631 \cos 29}$

Find the x and y components of v_C.

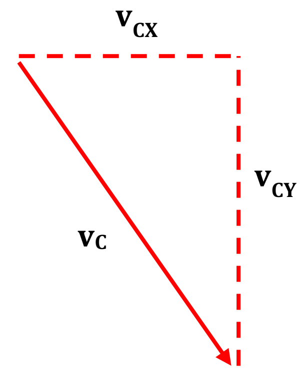
$v_{Cx} = a_x t + v_{BX}$
$\underline{v_{Cx} = 5.73631 \sin 29}$

Solve for the time in the y direction (needed to find v_{CY}).

$\Delta y_{BC} = \frac{1}{2}a_y t_{BC}^2 + v_{By}t_{BC}$
$-6.8 = \frac{1}{2}(-9.8)t_{BC}^2 + (5.74 \sin 29)\, t_{BC}$
$-6.8 = -4.9t_{BC}^2 + (2.78103)t_{BC}$
$0 = -4.9t_{BC}^2 + 2.78103t_{BC} + 6.8$
~~$t_{BC} = -0.928s$~~ or $\underline{t_{BC} = 1.49543\ s}$

$v_{Cy} = a_y t_{BC} + v_{By}$
$v_{Cy} = -9.8(1.49543) + 5.74 \cos 29$
$v_{Cy} = -14.6552 + 2.78102$
$\underline{v_{Cy} = -11.8742\ m/s}$

Find the net velocity at c using the x and y components.



$v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2}$
$v_C = \sqrt{(5.74 \cos 29)^2 + (-11.8742)^2}$
$\underline{v_C = -12.8906\ m/s}$

Calculate the Δx_{BC} using t_{BC} and v_{BX}.

$\Delta x_{BC} = \frac{1}{2}a_x t_{BC}^2 + v_{Bx}t_{BC}$
$\Delta x_{BC} = (5.74 \sin 29)\,(1.49543)$
$\underline{\Delta x_{BC} = 7.50272\ m}$

STAGE 4: Calculate the acceleration of the luge on the ground and μ_G between the luge and the ground after landing at x_c

Use Δx_{BC} and Δx_{BD} to find Δx_{CD}.

$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$
$\Delta x_{CD} = 75 - 7.50272$
$\underline{\Delta x_{CD} = 67.4973\ m}$

Calculate the acceleration of the luge on the ground using Δx_{CD}, v_D = 0 m/s, and the net velocity at C.

$v_D^2 = v_C^2 + 2a_G\Delta x_{CD}$
$0 = (-12.89)^2 + 2a_G(67.4973)$
$0 = 166.168 + 134.995a_G$
$134.995a_G = -166.168$
$\underline{a_G = -1.23092\ m/s^2}$

Write out the sum of forces in the various direction acting on the two objects and use F_{NG} to solve for μ_G.

$\sum F_{yG} : F_{NG} - F_{gG} = m_L a_{Gy}$
$F_{NG} - m_L g = 0$
$\underline{F_{NG} = m_L g}$

$\sum F_{xG} : -F_{FG} = m_L a_{Gx}$
$-\mu_G F_{NG} = m_L a_G$
$-\mu_G m_L g = m_L a_G$
$-\mu_G = \frac{m_L a_G}{m_L g}$
$-\mu_G = \frac{-1.23092}{9.8}$
$\boxed{\mu_G = 0.1256}$