# CS 453X: Class 25

Jacob Whitehill

# More on convolutional neural networks

# Convolution in 3-D

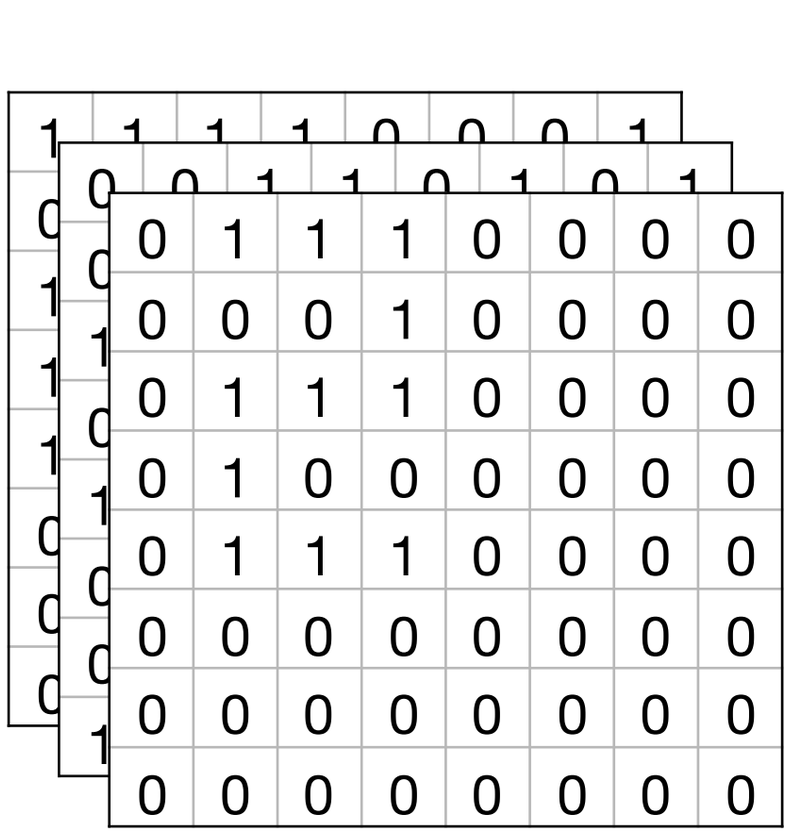
- It is possible to perform convolution in any number of dimensions.
- With neural networks, the usual formula of 3-D convolution is given by:

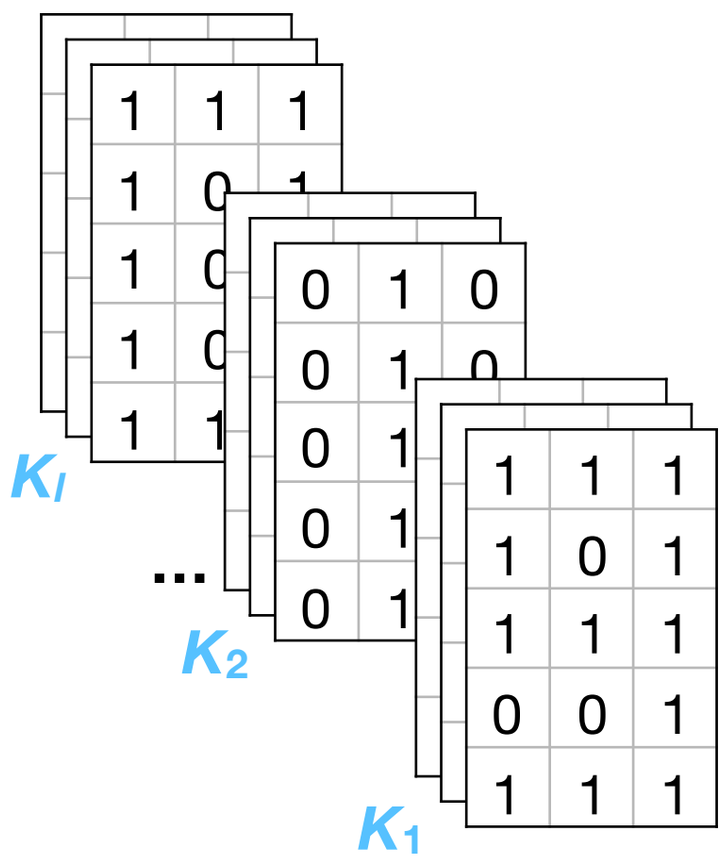
$$\mathrm{TM}(r,c) = \sum_{c=1}^{t_c} \sum_{i=-t_h/2}^{+t_h/2} \sum_{j=-t_w/2}^{+t_w/2} \mathrm{im}[r+i, c+j, c]\mathrm{t}[i,j,c]$$

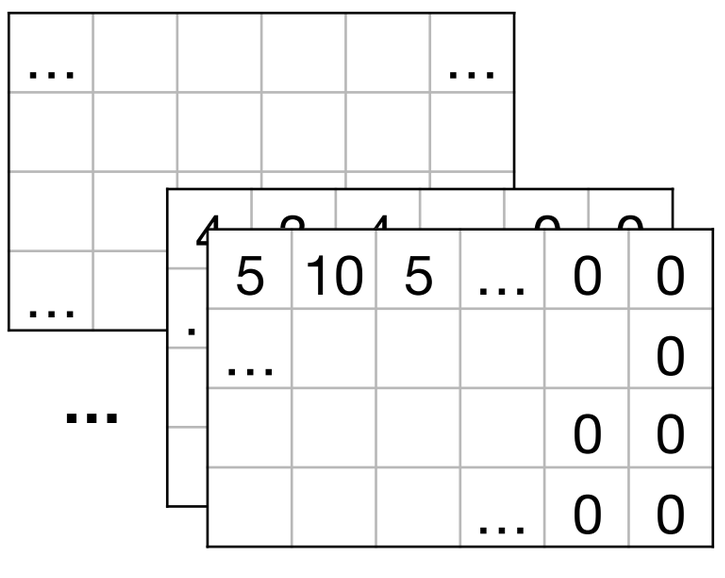
where $t_c$ is the number of **channels** in the convolution kernel (and also the input image).

# Convolution: multiple output channels

- By applying multiple convolution kernels to each input image, we can produce multiple feature maps (recall the MNIST example):

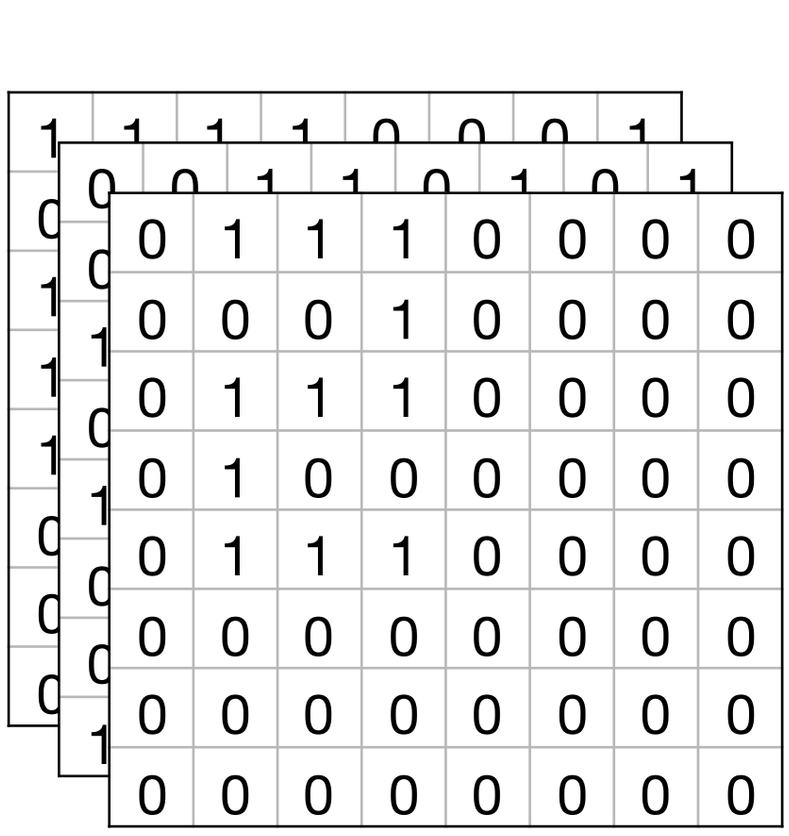


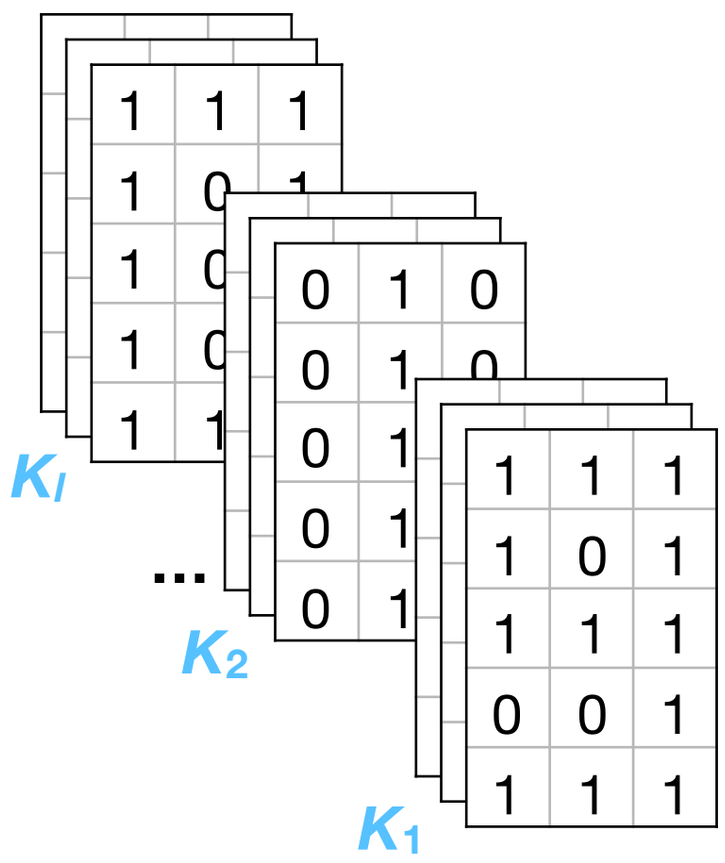



# Convolution: multiple output channels

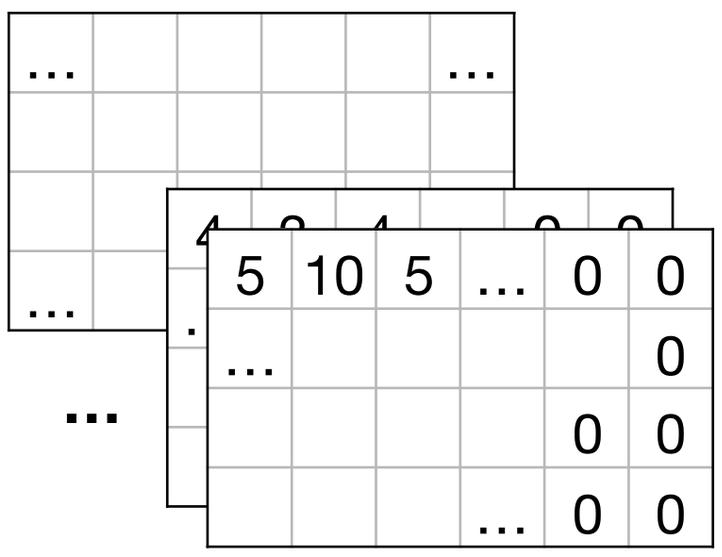
- Hence, convolution can take a multi-channel image as *input*, and also produce a multi-channel image as *output*.




Image




Multiple kernels




Multiple feature maps

# Convolutional neural networks

# Convolutional neural networks

- Based on this infrastructure, we can construct **convolutional neural networks** (**CNNs**) — networks that consist of 1+ convolutional layers (and usually some non-convolutional layers too).

- CNNs have revolutionized computer vision, especially in object detection, object recognition, semantic segmentation, activity recognition, and other tasks.

# Convolutional neural networks

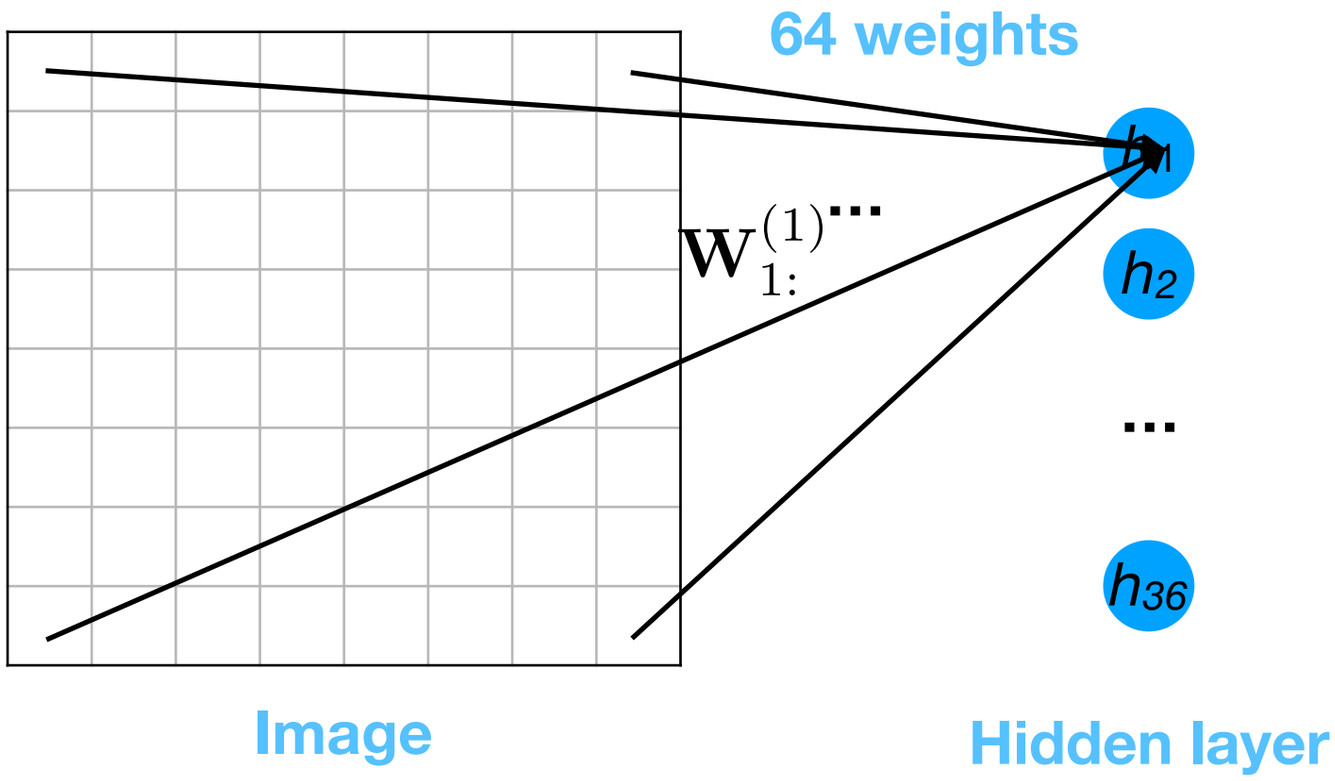
- While the kernels in the examples so far were built by hand, this is almost never done in practice.

- Instead, we train the weights using back-propagation, just like with feed-forward neural networks.

- With CNNs, the learned weights represent the elements of the convolution kernels.

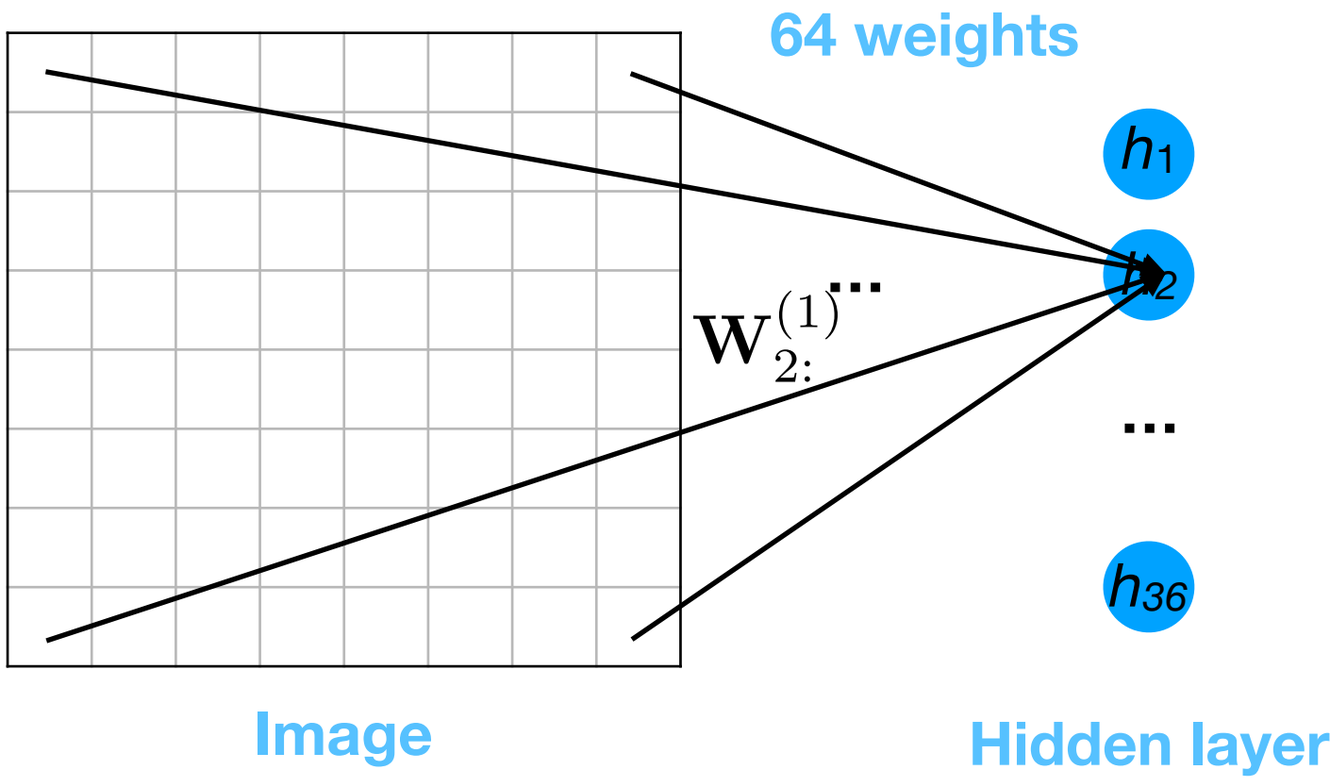
# Convolutional neural networks

- CNNs are a special case of feed-forward (FF) NNs.
- In the FF NN below, each of the 36 hidden units is associated with 64 weights.



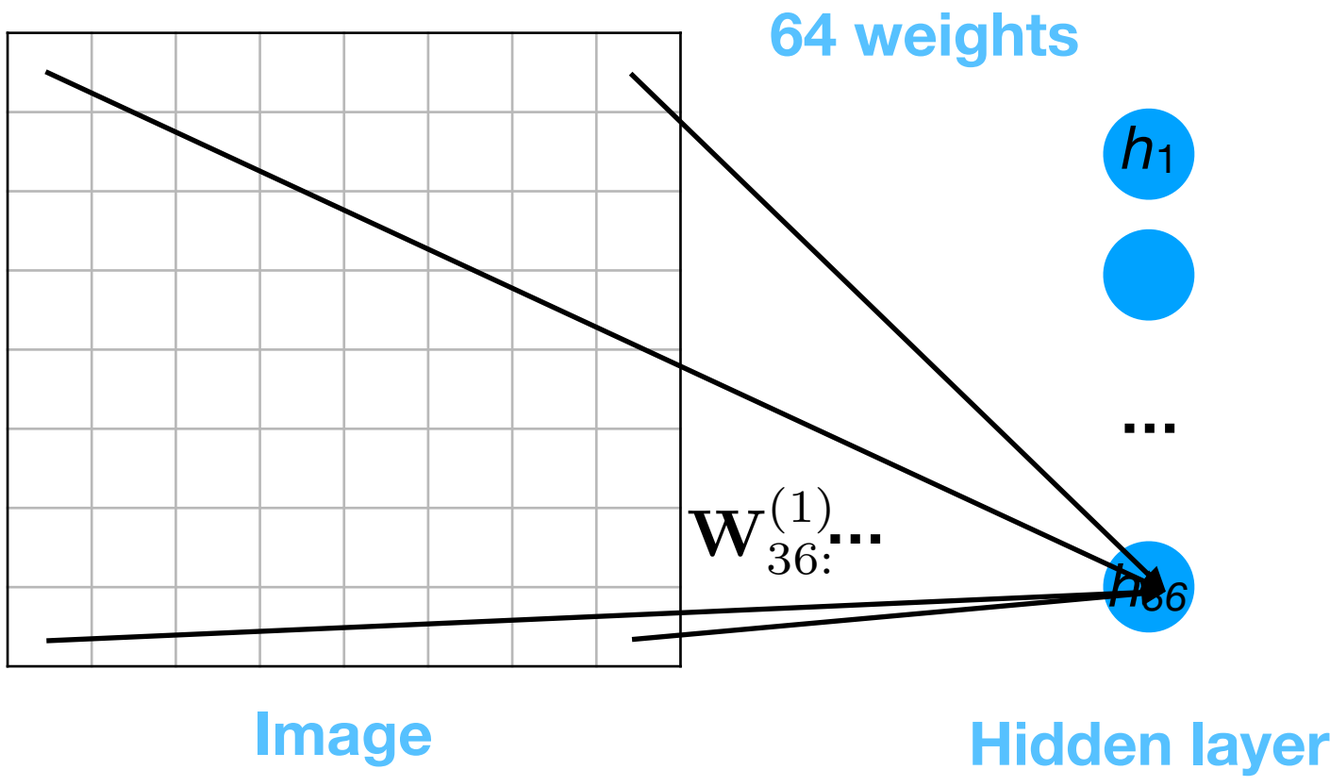
# Convolutional neural networks

- CNNs are a special case of feed-forward (FF) NNs.
- In the FF NN below, each of the 36 hidden units is associated with 64 weights.



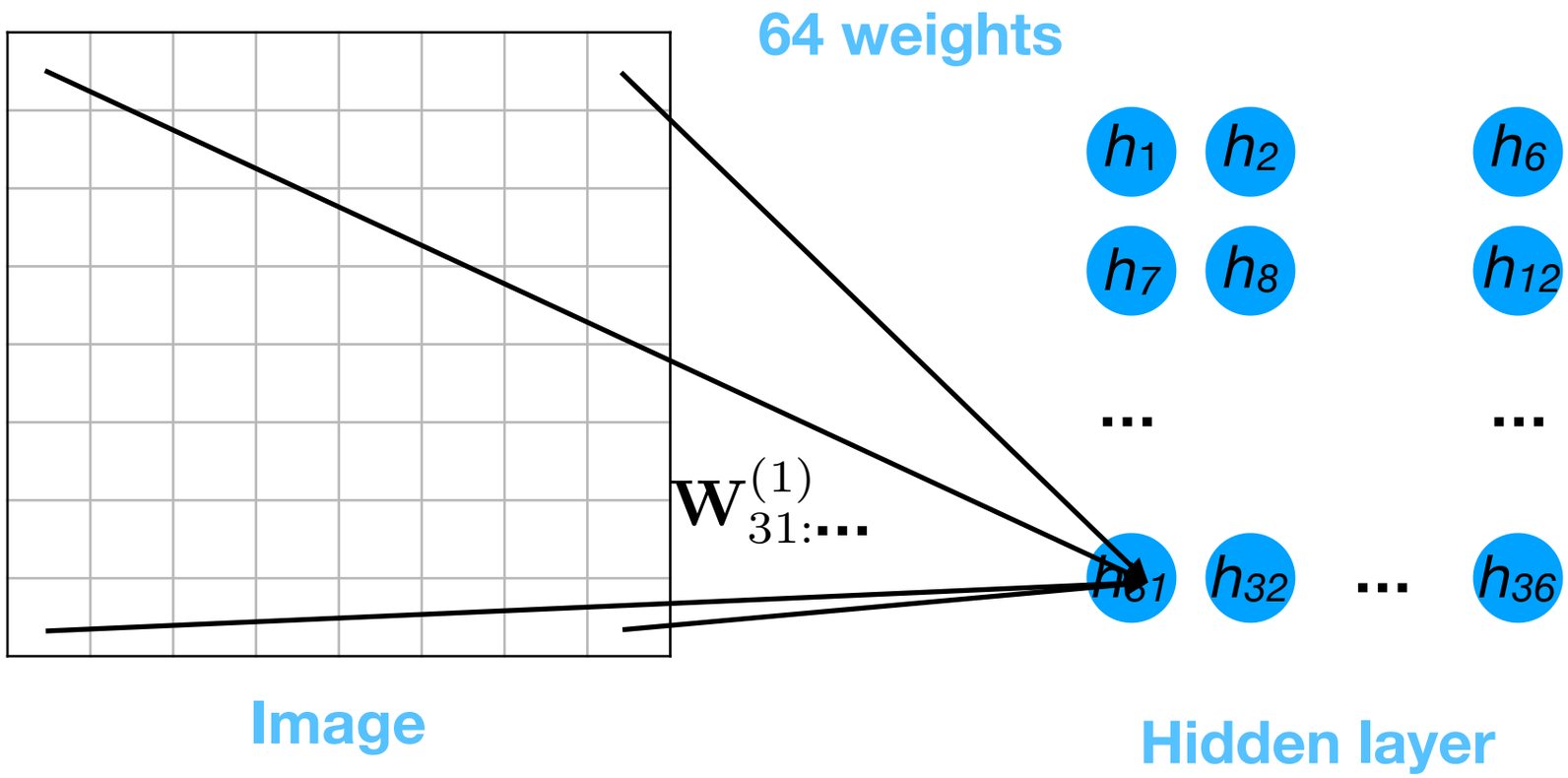
# Convolutional neural networks

- CNNs are a special case of feed-forward (FF) NNs.
- In the FF NN below, each of the 36 hidden units is associated with 64 weights.



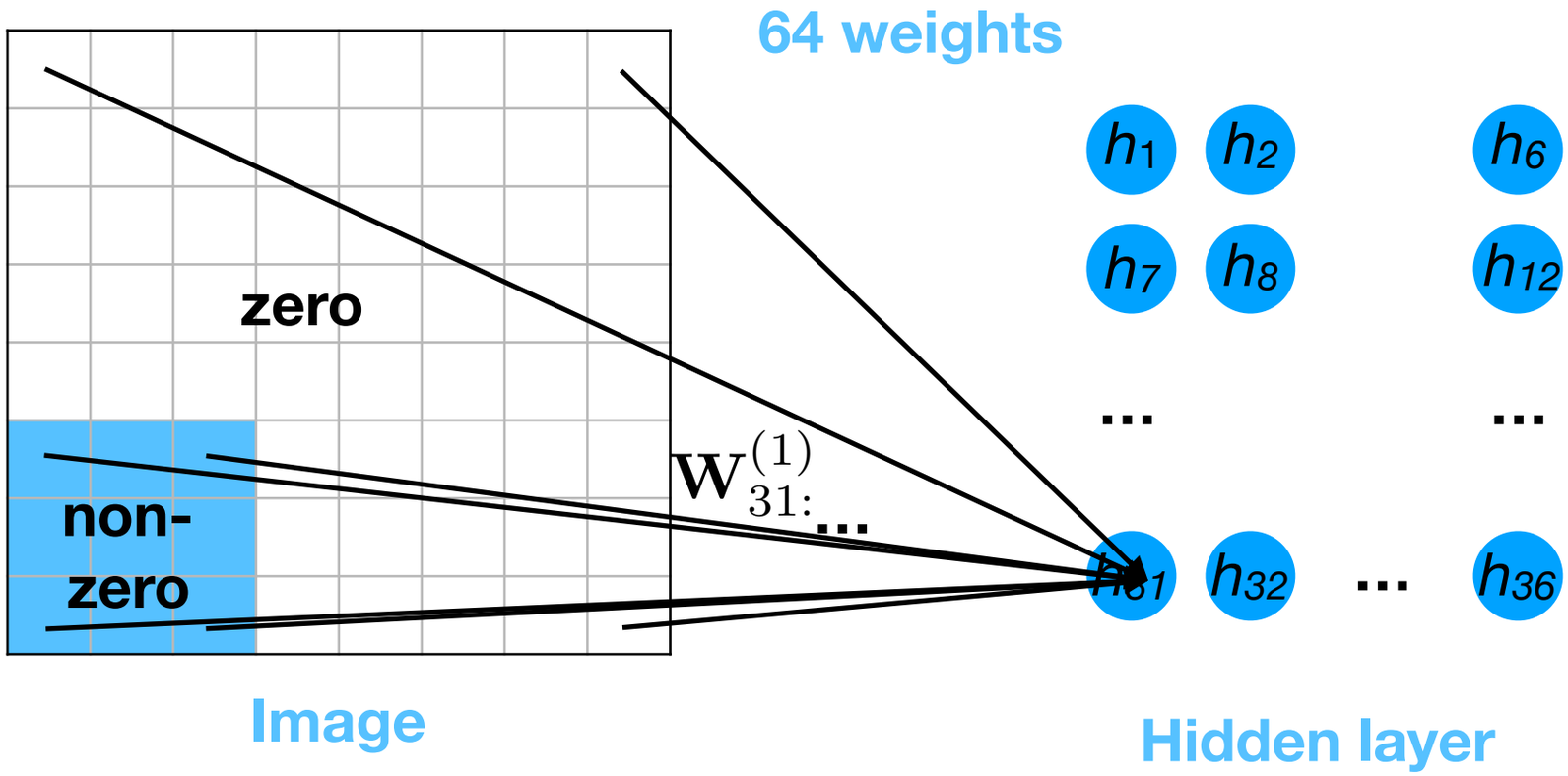
# Convolutional neural networks

- We can re-arrange the hidden units into a grid (this just affects the visualization, not the computation).



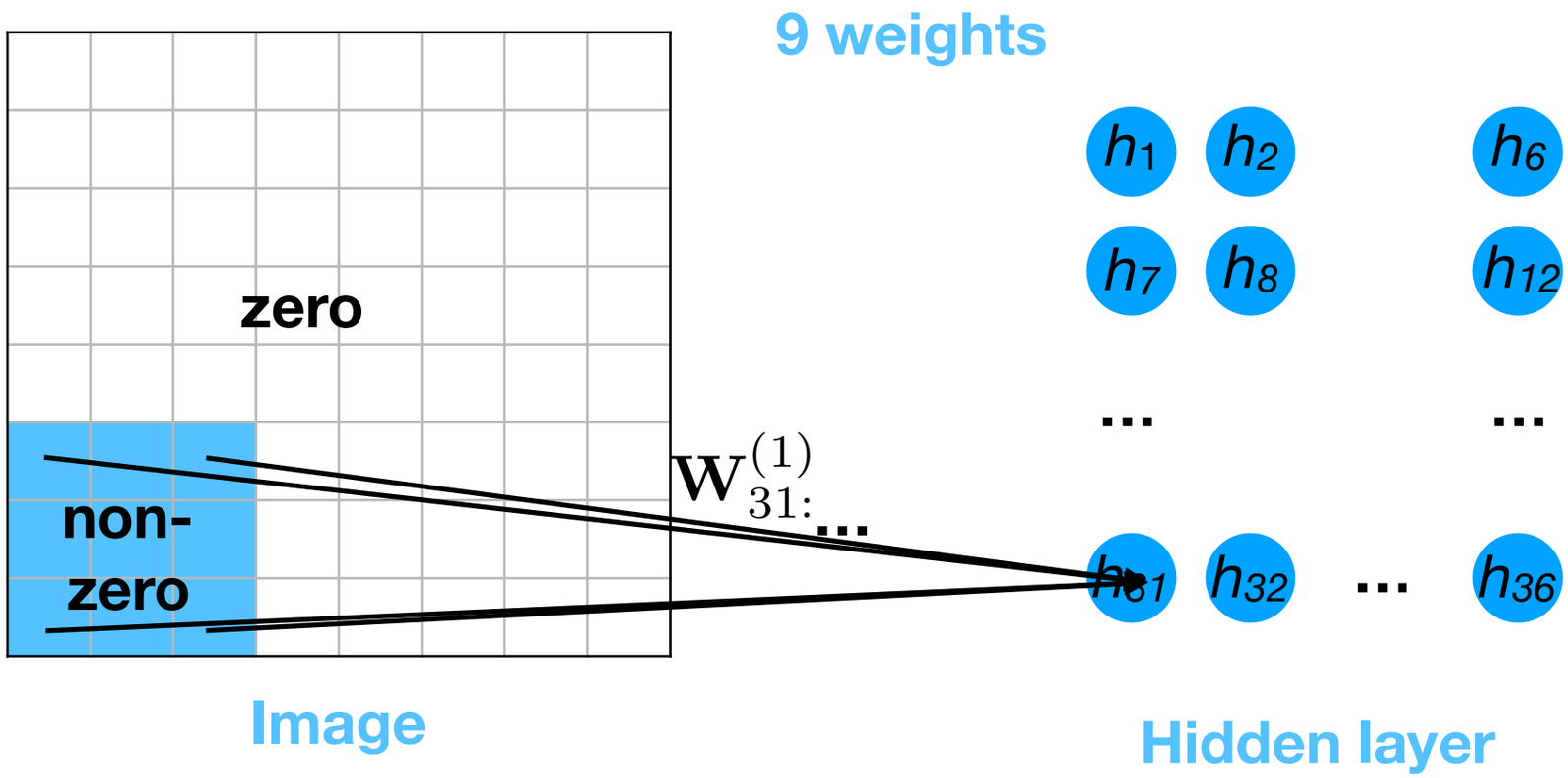
# Convolutional neural networks

- We can also require that most of the weights for each hidden unit be 0.



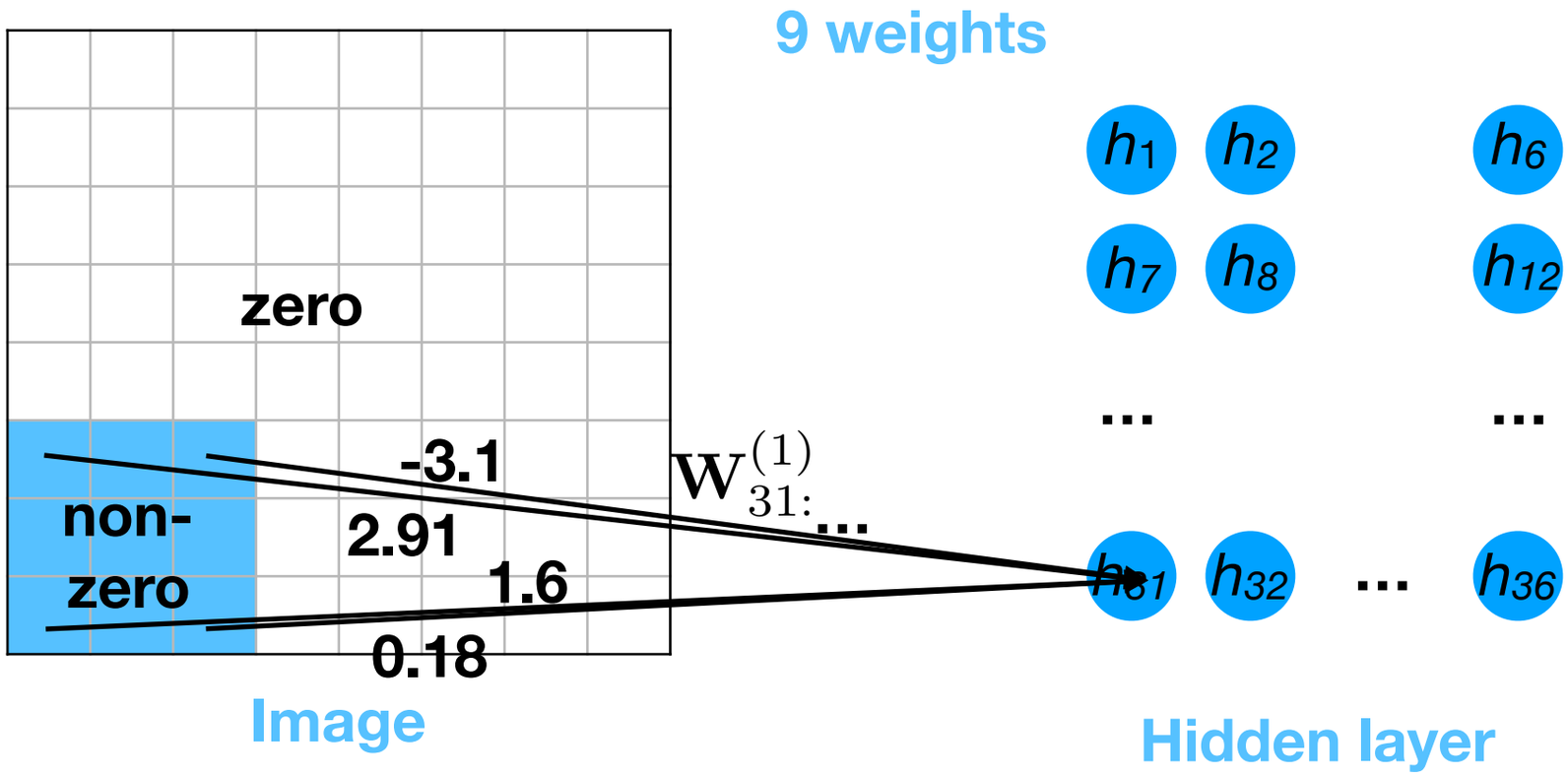
# Convolutional neural networks

- Since the image pixels corresponding to the 0-weights contribute nothing to each hidden unit, they can be removed, resulting in just 9 weights per hidden unit.



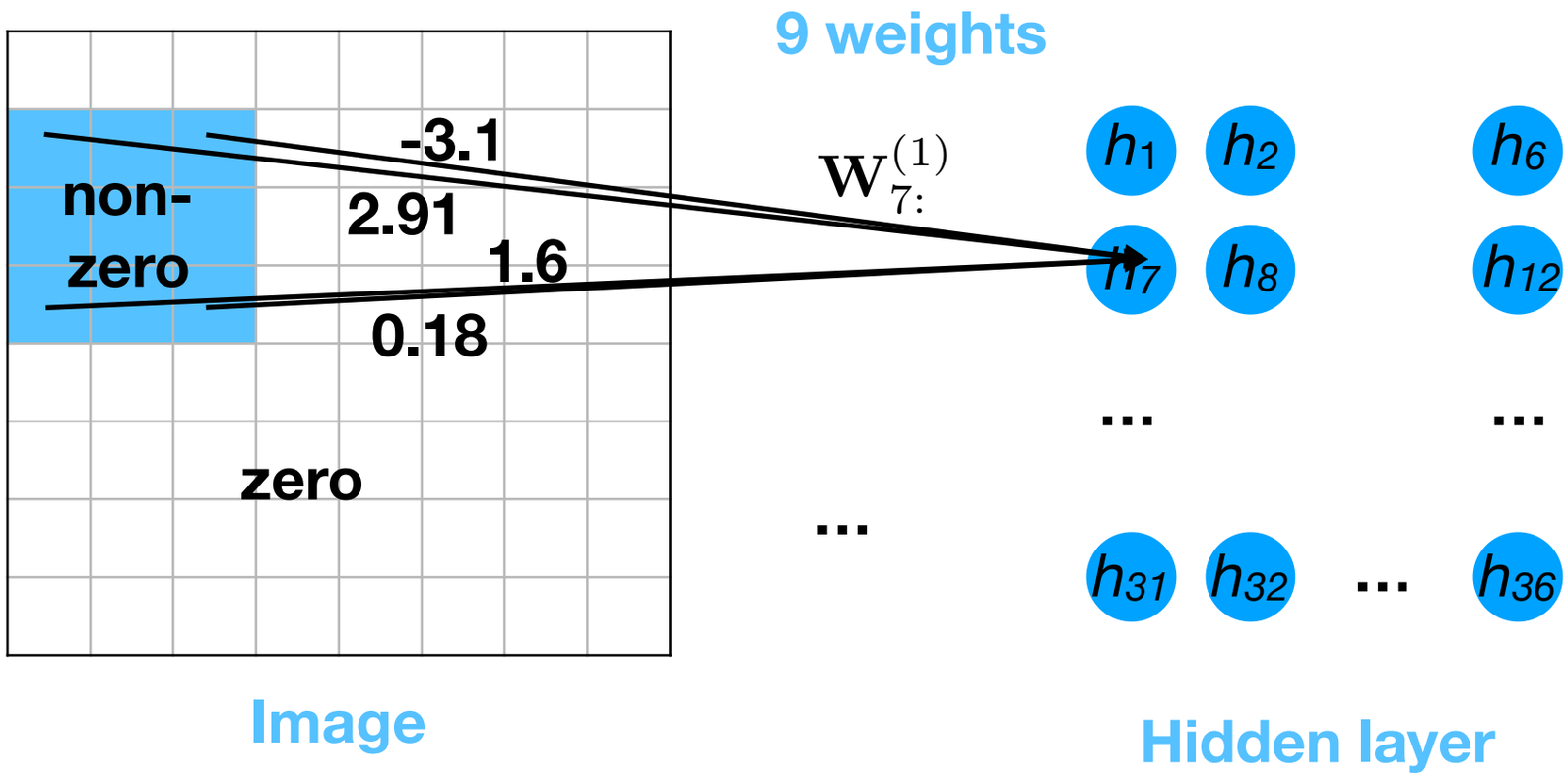
# Convolutional neural networks

- Can can further constrain weight matrix $\mathbf{W}^{(1)}$ by requiring that each 3x3 set of non-zero weights be the *same* for each of the hidden units.



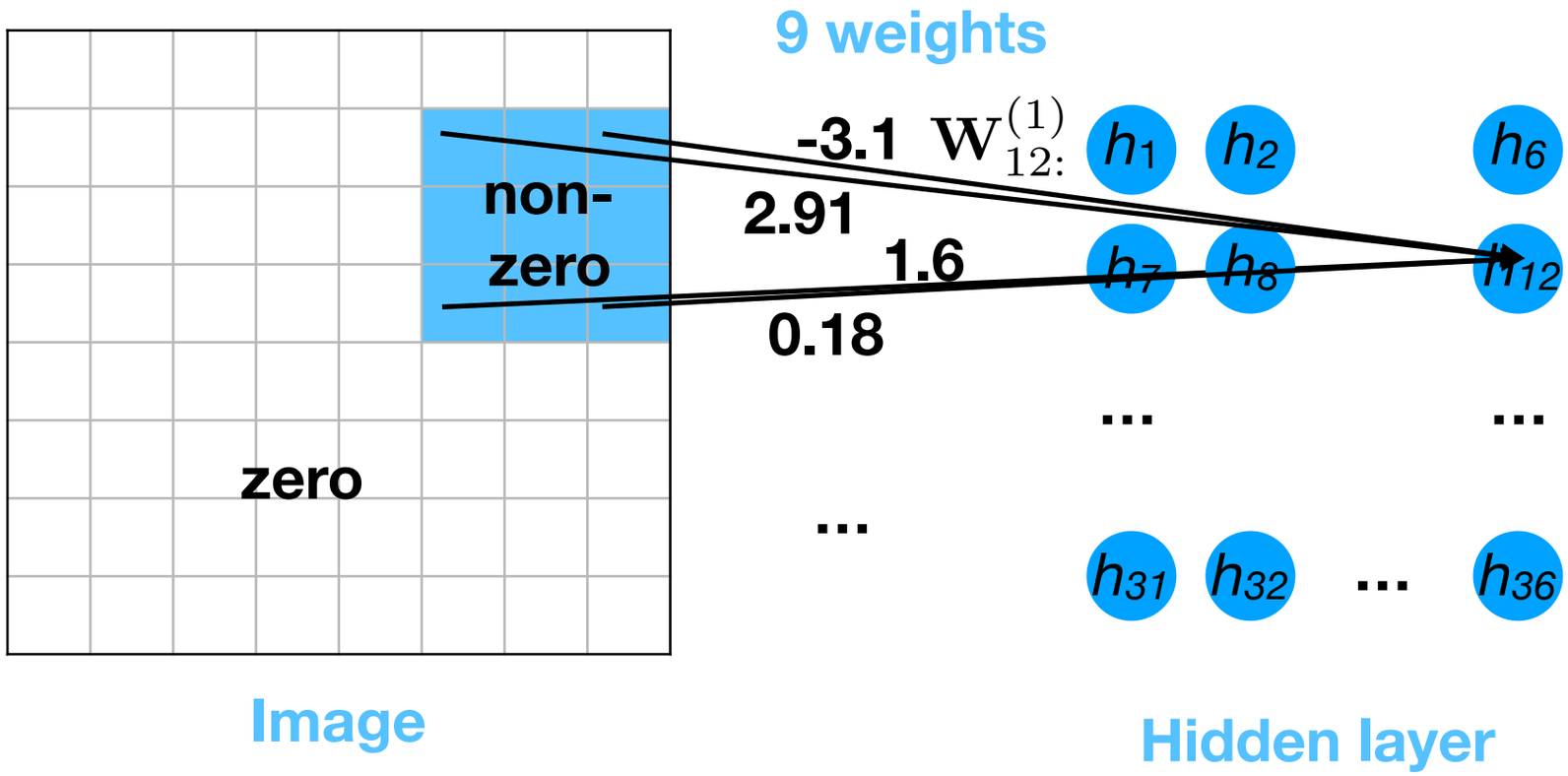
# Convolutional neural networks

- Can can further constrain weight matrix $\mathbf{W}^{(1)}$ by requiring that each 3x3 set of non-zero weights be the *same* for each of the hidden units.



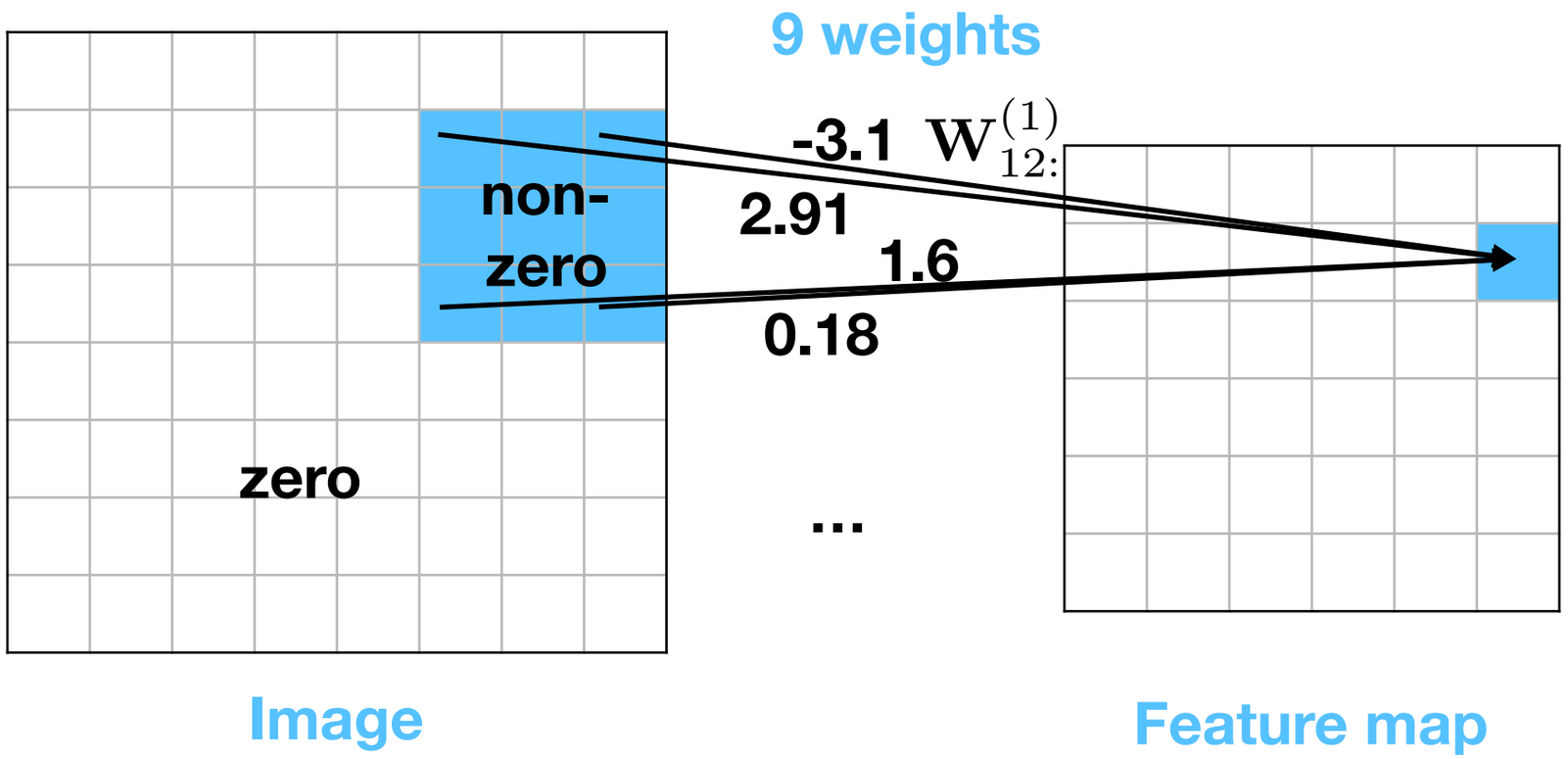
# Convolutional neural networks

- Can can further constrain weight matrix $\mathbf{W}^{(1)}$ by requiring that each 3x3 set of non-zero weights be the *same* for each of the hidden units.



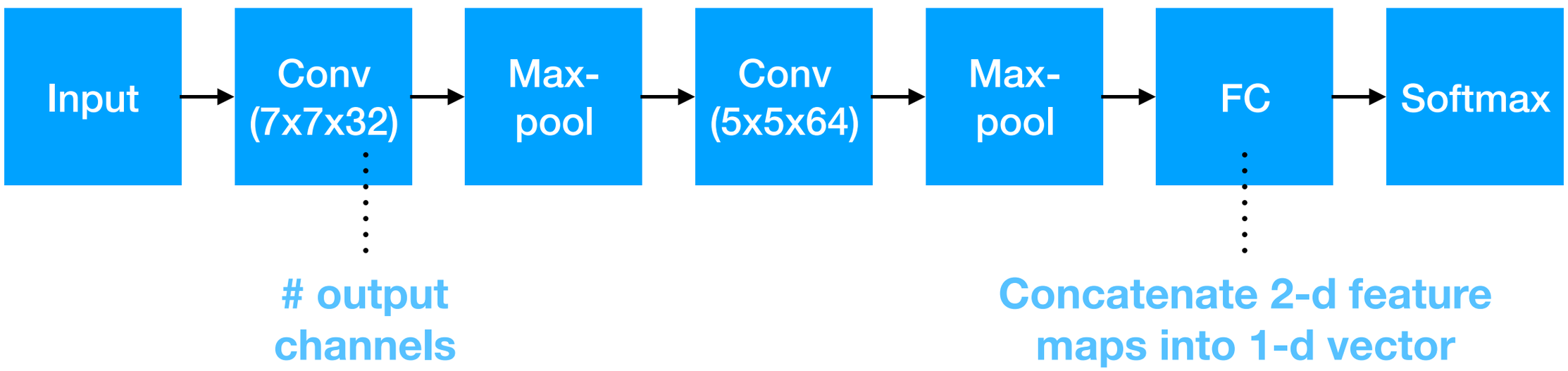
# Convolutional neural networks

- This is now completely equivalent to a convolutional layer instead of a general feed-forward layer.



# CNN architecture

- CNNs (since ~2012) typically employ:
  - Multiple convolutional layers
  - Pooling layers (e.g., max-pool) between some of the convolutional layers.
  - Feed-forward layers at the end.
  - Non-linear activation functions between each layer.
- Example:



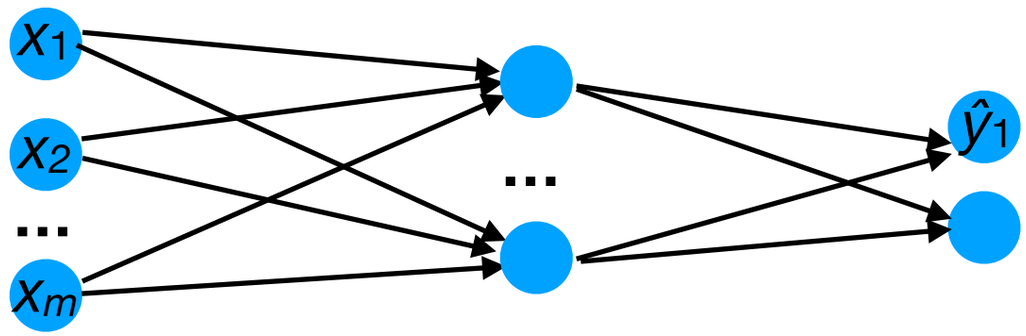
# CNN architecture

- Trend (~2016+):
  - Less pooling
  - More convolutional layers

# Recurrent neural networks

# Fixed length versus variable length

- Up till now, all the ML models we have considered have processed inputs **x** of some **fixed length** *m* to predict some target value *y*.




- But what if *m* varies from example to example?

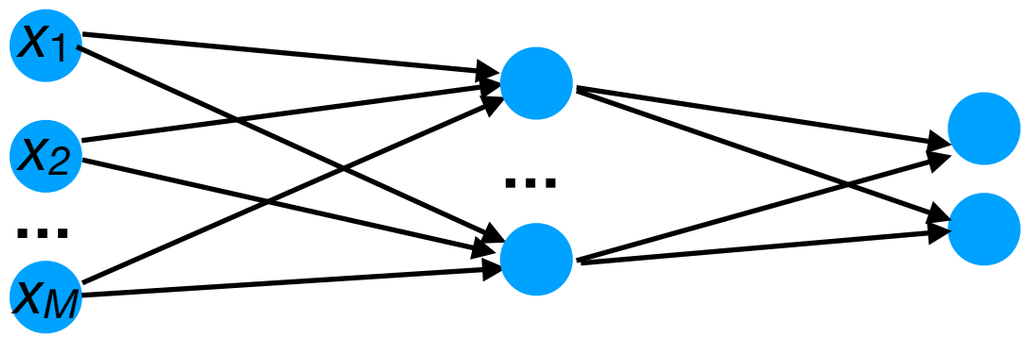
# Fixed length versus variable length

- Examples of **variable-length** inputs:
  - Text passages: number of words.
  - Videos: number of frames.
  - Time series: number of events.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

    1. Decide what the maximum length should be, and "pad" the input to always reach that length.




E.g., if max-length $M$=6, but actual length of some example $m$=4, then pad with 2 zeros.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

    1. Decide what the maximum length should be, and "pad" the input to always reach that length.

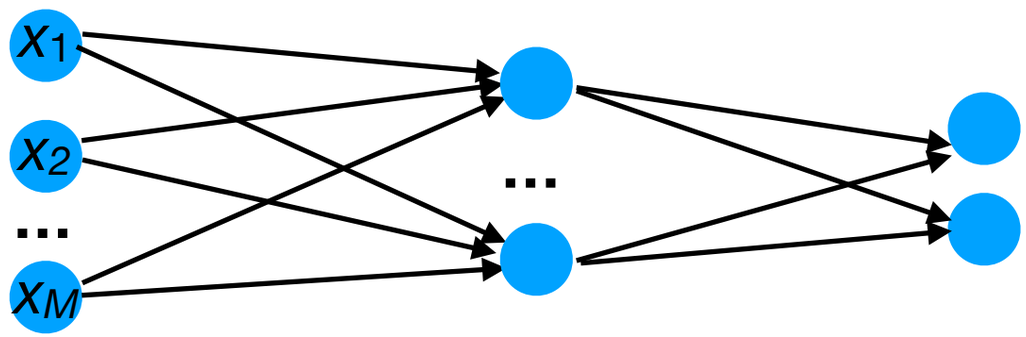
    - Advantages:
        - Simple to implement.

    - Disadvantages:
        - Wasteful of memory
        - Hard to learn weights from input units close to *M*.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

2. Decide what the minimum length should be, and discard any features beyond this length.




E.g., if min-length $M$=6, but actual length of some example $m$=16, then ignore the last 12 features.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

    2. Decide what the minimum length should be, and discard any features beyond this length.

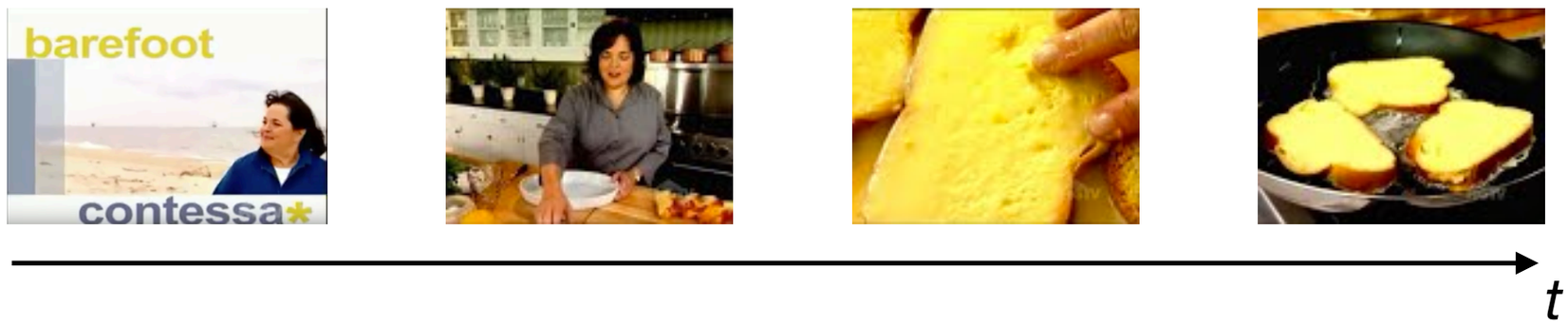
- Advantages:
    - Simple to implement.

- Disadvantages:
    - Throws away potentially valuable information.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

3. Extract summary features from the sequence that do not depend on the input length.

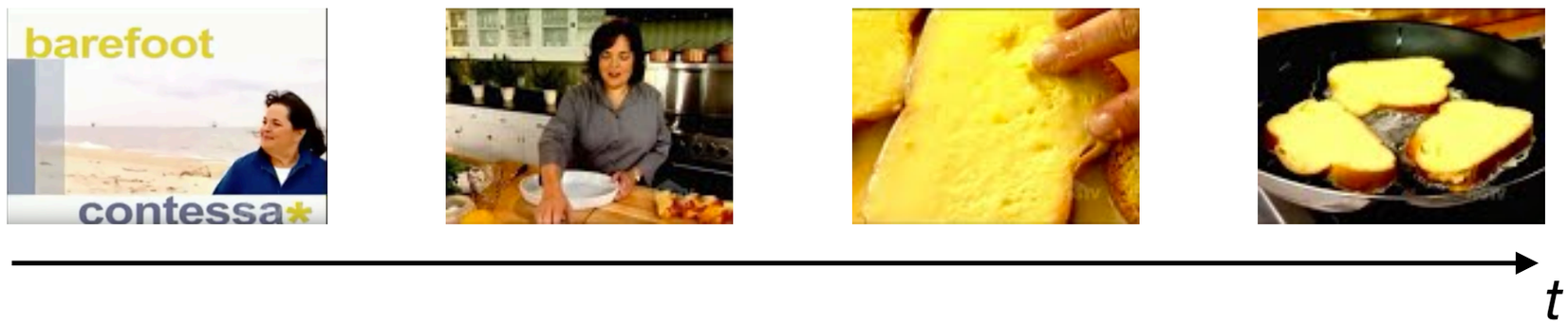



E.g., extract aggregate color information, number of detected objects of fixed set of classes, etc.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

3. Extract summary features from the sequence that do not depend on the input length.




Judge the type of video based on the fixed-length feature representation.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

    3. Extract summary features from the sequence that do not depend on the input length.

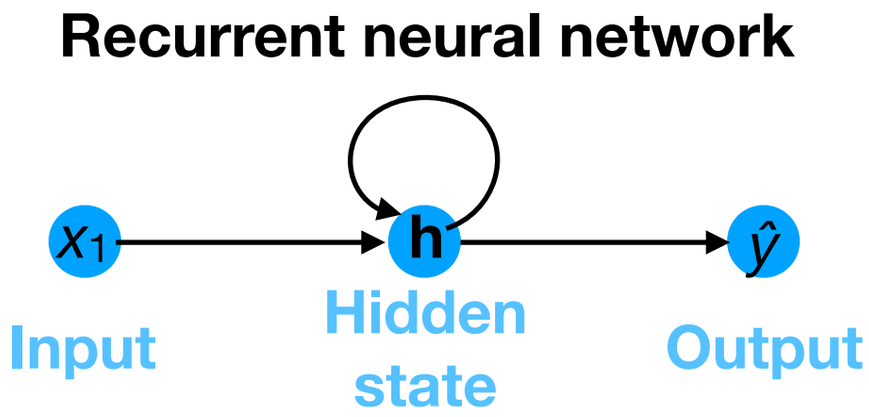
- Advantages:
    - Simple to implement; can give high accuracy.

- Disadvantages:
    - Requires manual feature engineering.

# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

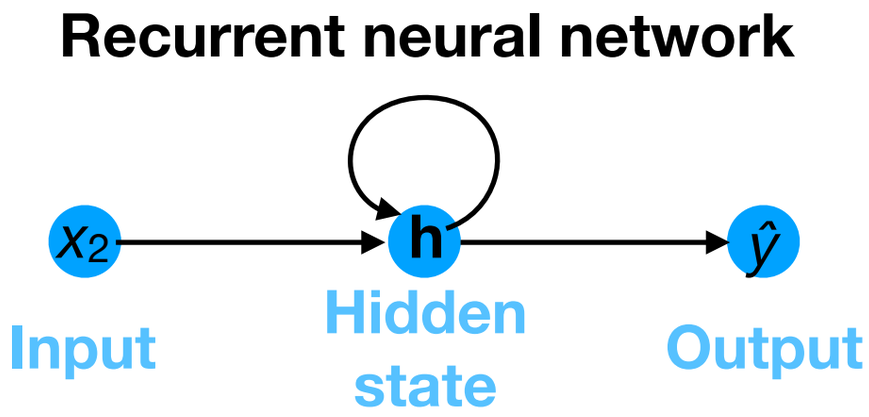
    4. Scan the input in blocks of *k* features at a time, and update a “hidden state” to the store most important information.



# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

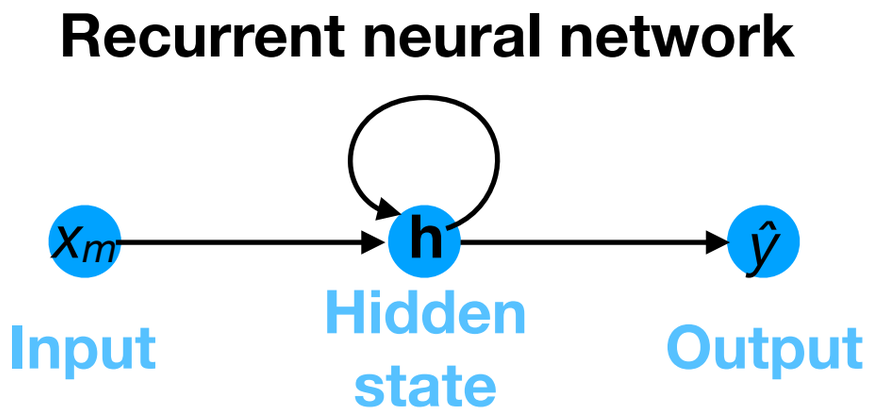
    4. Scan the input in blocks of *k* features at a time, and update a “hidden state” to the store most important information.



# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

    4. Scan the input in blocks of *k* features at a time, and update a “hidden state” to the store most important information.



# Handling variable-length inputs

- Possible strategies for handling variable-length inputs:

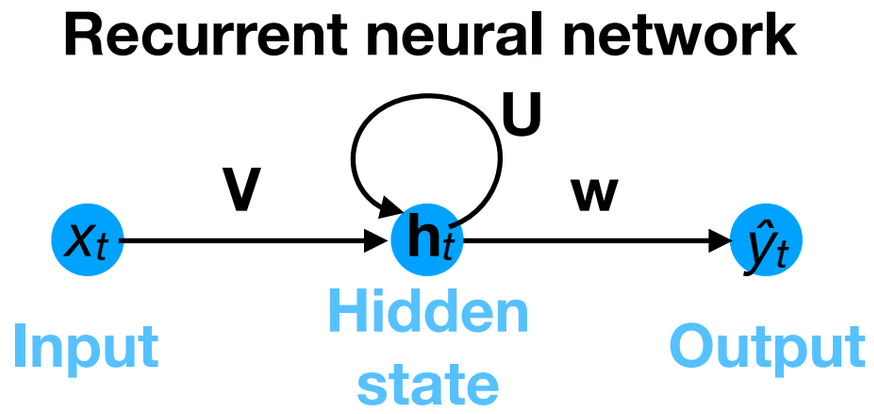
    3. Scan the input one feature at a time, and update a "hidden state" to the store most important information.

- Advantages:
    - More powerful, often resulting in higher accuracy.

- Disadvantages:
    - More complicated to implement & train.

# Recurrent neural network

- We can construct a minimal **recurrent neural network** (RNN) as follows:




$$\hat{y}_t = g(\mathbf{x}_t; \mathbf{U}, \mathbf{V}, \mathbf{w}) = \mathbf{h}_t^\top \mathbf{w}$$

$$\mathbf{h}_t = \sigma\left(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{V}\mathbf{x}_t\right)$$