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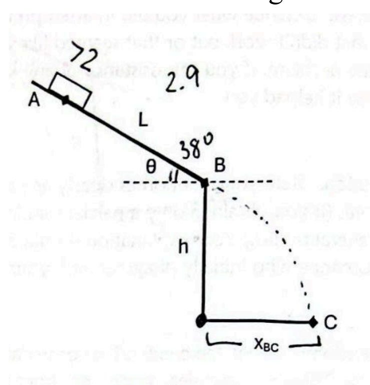
Problem Statement:
A puck, with a mass of 72g, travels down a 2.9m ramp at an angle of 38° after starting at rest. The coefficient between the ramp and the puck is 0.17. At the bottom of the ramp is a 1.6m tall counter going straight down. How far away does the puck land from the base of the counter? Make an algorithm that calculates the angle of the ramp that will result in it traveling the farthest.



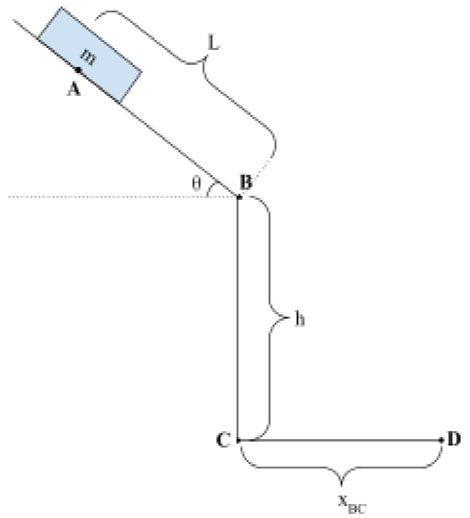
Process:

a) We started by solving part A individually and then comparing our results. Originally, our answers did not match up, so we compared our work and identified any variables left out of the equations used to solve this problem. The first part involved drawing a Free Body Diagram of the puck on an inclined plane and considering all the forces acting on it to find its acceleration down the ramp. We used the acceleration found to calculate the final velocity of the puck once it reaches the end of the ramp. After leaving the ramp, the puck becomes a projectile. In the projectile phase, we had to split the velocity from the first section into x and y components and then used a system of equations–one equation describing motion in the x direction, the other describing motion in the y direction–in order to find the distance the puck would land from the base of the counter where the projectile started.
It took us several attempts to solve this problem. In the first one, we forgot to include gravity when solving for the projectile part. The second time, we used the wrong theta value, forgetting to take into consideration that the angle was negative. The third time, we forgot that v_x was the x component of v, using the full velocity, 5.23, instead of 5.23cos(38°). We were able to discover all these small errors when comparing each other's work.

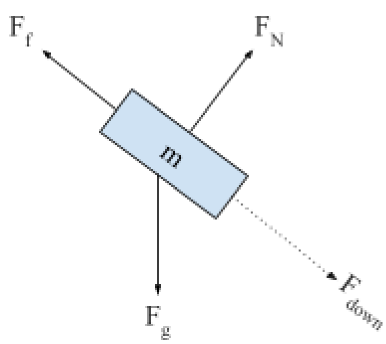
b) When solving for this, we first thought we could represent the angle in terms of a function. We used Desmos Graphing Calculator to create a graph relating an angle to the resulting distance. We were then able to find the maximum distance. When we got this answer, we wanted to ensure that the answer was correct. In order to do this we thought we could use a spreadsheet to input different angle values, proving that our function worked. We represented each part of the problem as an algorithm within the spreadsheet. At first, we attempted to write the algorithm within one line. This proved to be difficult, so we broke it down into multiple steps, first converting degrees to radians, then finding acceleration, velocity, time, and then, finally, the distance. We found the optimal whole angle, then considered tenths of angles in the optimal range, followed by hundredths of angles, confirming the angle measure we found using Desmos.

Solution:

a) We will refer to the following diagram for variables related to this problem:



Here, m refers to the mass of the puck, L refers to the length of the ramp (from the puck to the bottom of the ramp), θ is the angle of the ramp above the horizontal, h is the height from the bottom of the ramp to the ground, and x_{BC} is the distance that the puck lands from point **C** after sliding down the ramp. We can calculate x_{BC} by first considering the acceleration of the puck while on the ramp. For this, we first found the net force on the puck at its initial point. Below is a free-body diagram of all forces on the ramp:



The force F_{down} is drawn with a dotted line because it is not a distinct force acting on the puck, it is just the component of F_g in the direction down the ramp. The net force on the puck would be F_{down} - F_f, because the normal force F_N matches the corresponding component of gravity that goes in the direction opposite of it. First, we can find F_g as mg, where m is given as 0.072 kg and g is known to be 9.8 m/s^2. Substituting these values, F_g = 0.072 kg × 9.8 m/s^2 = 0.0756 N. F_{down} is $F_g\sin(\theta)$ because it is the component of gravity down the ramp. θ is given to be 38°, so F_{down} = 0.0756 N × sin(38°) = 0.4344 N. Next, we can find F_N, which should be $F_g\cos(\theta)$. Substituting once again, we get F_N = 0.0756 N × cos(38°) = 0.5560 N. This lets us solve for F_f, which is equal to μF_N where μ is the coefficient of friction between the ramp and the puck. The value for μ is given as 0.17. Substituting, we get $F_f = 0.17 \times 0.5560$ N = 0.0945 N. We can finally find the net force on the puck, $F_{down} - F_f$. Substituting our values into the equation, we get F_{net} = 0.4344 N - 0.0945 N = 0.3399 N. To find the puck's acceleration down the ramp, we can use the law $F_{net} = ma$. Rearranging this yields $a = F_{net} / m$ = 0.3399 N / 0.072 kg = 4.7207 m/s^2.

Next, we can aim to find the velocity of the puck at the end of the ramp, which we will call v. To find this, we can consider that the puck starts from rest, at 0 m/s (we will call this starting speed v_0), and we can use the equation $v^2 = v_0^2 + 2a\Delta x$. The displacement Δx to the end of the ramp is L, which is given as 2.9 m. Also, since v_0 = 0 m/s, we know v_0^2 = 0 m^2/s^2, so we can disregard this term. This gives us $v^2 = 2aL$. Substituting, we get $v^2 = 2 \times 4.7207$ m/s^2 $\times$ 2.9 m = 27.3798 m^2/s^2, and since v would be the square root of this value, we find v = 5.2326 m/s.

The final part of our problem involves projectile motion, as the puck is in freefall with our velocity v after sliding off the ramp. Our velocity can be split into horizontal and vertical components v_x and v_y. Since v is in the downward direction at an angle θ = 38°, v_x would be vcos(38°) and v_y would be vsin(38°). Substituting, we have v_x = 5.2326 m/s × cos(38°) = 4.1233 m/s. Similarly, we find v_y as 5.2326 m/s × sin(38°) = 3.2215 m/s. Next, we can find the distance x_{BC} as $v_x t$ where t is the time that the puck is in projectile motion. We can use the kinematics equation $h = v_y t + \frac{1}{2}gt^2$ to solve for t using the quadratic formula on the rearranged equation $\frac{1}{2}gt^2 + v_y t - h = 0$, using substituted coefficients (where h is given as 1.6m) like so: ½ × 9.8 m/s^2 × t^2 + 3.2215 m/s × t - 1.6 m. We consider the positive root as the negative root would imply negative time, which is not plausible. This gives us t = 0.3305 s. Substituting our time into $x_{BC} = v_x t$, we get x_{BC} = 4.1233 m/s × 0.3305 s = 1.3628m.

b) We found that in general, the greatest possible x_{BC} in terms of the ramp angle θ is 1.5178m when θ = 26.65. We checked this answer through both Desmos and a spreadsheet. Using Desmos Graphing Calculator, we parameterized all values solved for in part (a) as functions of θ, as pictured below:

$$F_g = mg$$

$$F_x(\theta) = F_g \sin(\theta)$$

$$F_N(\theta) = F_g \cos(\theta)$$

$$F_f(\theta) = \mu F_N(\theta)$$

$$F(\theta) = F_x(\theta) - F_f(\theta)$$

$$a(\theta) = \frac{F(\theta)}{m}$$

$$v_f(\theta) = \sqrt{2a(\theta)L}$$

$$v_x(\theta) = v_f(\theta)\cos(\theta)$$

$$v_y(\theta) = v_f(\theta)\sin(\theta)$$

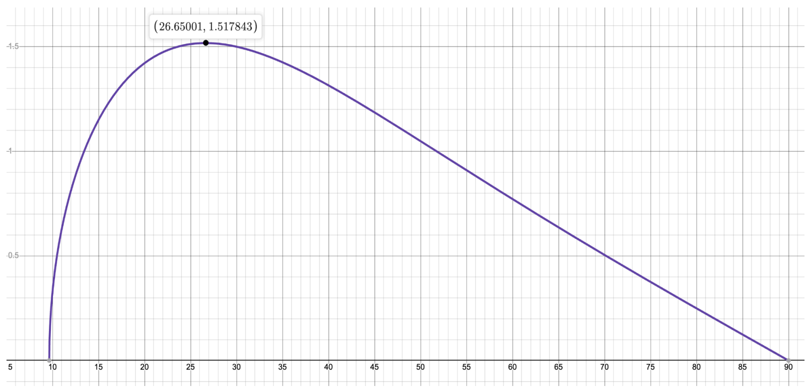
Though the formula for time may appear complicated, it is just the quadratic formula's positive solution.

$$t(\theta) = \frac{-v_y(\theta) + \sqrt{\left(v_y(\theta)\right)^2 - 4\left(\frac{g}{2}\right)(-h)}}{2\left(\frac{g}{2}\right)}$$

We then plotted the function for x_{BC} as $f(\theta)$:

$$f(\theta) = t(\theta)v_x(\theta)$$

The plot for this function yielded the graph below, with a maxima at $\theta = 26.65^{o}$ where $f(\theta) =$ 1.51784.



We also checked this answer using a spreadsheet. We considered every integer angle 0-90^{o} and found that 27^{o} was the most optimal whole number angle, sending the puck 1.5176351m from the base of the counter. We determined that the most exact optimal angle lies somewhere between 26^{o} and 27^{o} and then considered every angle from 26-27^{o} at an increment of 0.1^{o}. We found that the greatest distance from the base of the counter was 1.5178388m, the distance when the ramp was 26.6^{o} and 26.7^{o}. The optimal angle appears to be directly in the center of 26.6^{o} and 26.7^{o}.

Extensions:

- How would the distance from the counter change if the ramp was frictionless? What would the new angle that maximizes the distance be?
- How would the distance from the counter change if the puck didn't start at rest, and instead started with a velocity of 0.5 m/s? What about if it started with a velocity of -0.5m/s, starting at the bottom of the ramp and traveling up the ramp at first?