Case Study: Weir System Calibration

Introduction

This case study was developed by mechanical engineering major Jay Keist as part of his Interactive Qualifying Project at WPI. As a participant in the WPI cooperative education program, Jay had worked for a semester for a supplier of equipment to the paper industry. The engineering and statistical aspects of the case study draw heavily on Jay's experiences with a similar project at that company. A particular strength of this case study lies in its realistic portrayal of (1) how engineering and statistics interact and (2) the iterative nature of industrial problem-solving.

The statistical methods used in this case study include confidence intervals, tolerance intervals, and simple linear regression.

The Case Study

The R&D Meeting

At the research and development meeting, Al Showman was updating several engineers on the progress of the potential new product for Web Technologies. Web Technologies had manufactured various products for the paper industry for the past fifty years. Over those fifty years, the company had made some close ties with various paper plants around the Northeast.

Al was head of research and development, and he had over ten years experience with the paper industry. Al had been working on the new dual doctor system for over a year now.

"The prototype dual doctor has been made and we have a mill interested in letting us try the dual doctor on one of their paper machines. Now we need to obtain some data on the water removal rate of the existing doctor and be able to compare it with the water removal rate of the dual doctor system."

The dual doctor system the engineers had been working on consisted of two doctor blades and doctor holders in contact on one roll. The doctor system now in use consisted of one doctor blade with a holder. The doctor blade is a long, thin, rectangular blade that contacts the entire length of the paper roll (around 400 inches). The paper runs between the doctor blade and the roll surface. The paper at this section of the paper machine contains a large amount of water. Before the paper reaches the dryer section of the paper machine, it is advantageous to remove as much water from the paper as possible. The doctor system removes water from the paper by exerting a pressure on the paper as it passes beneath the blade. With two doctor blades in succession, the engineers were hoping for at least a 50% increase in water removal as compared to the existing doctor system.

Al continued, "Now, Bill has been working on a weir system which will be able to measure the water removal rates from the doctor system." Bill Radky had also had several years experience in the paper industry; however, he had just recently moved into research and development. He was working on several projects and finding time to work on his weir system was difficult.

A weir system is an overflow structure where the flow rate is a unique function of the depth, or head, of the water above the weir crest. A weir system includes the weir plate, an approach channel, and a depth measuring device (Figure 1). The weir crest is the bottom of the overflow section on the weir plate (notch). Thin plate weirs are good for accurate flow measurement, and the weir setup is simple and reliable. The weir system built for this application used a 45 degree triangular weir overflow section. Bill had the weir system made with stainless steel sheet metal and the weir channel had dimensions around $2 \times 2 \times 6$ feet.

Bill explained, "Yeah, the weir system is built and it should be able to fit easily enough under the catch basin from the weir system. The weir system was made as close as possible to ASTM (American Standards of Testing and Materials) standards for flow measurement with thin plate weirs. However, we have limited space at the mill, and we had to make the channel smaller than the dimensions stipulated in ASTM. I will have the weir calibrated at the civil hydrology laboratory. There they have the equipment to run various known flow rates through the weir. We can then take several channel depths at various

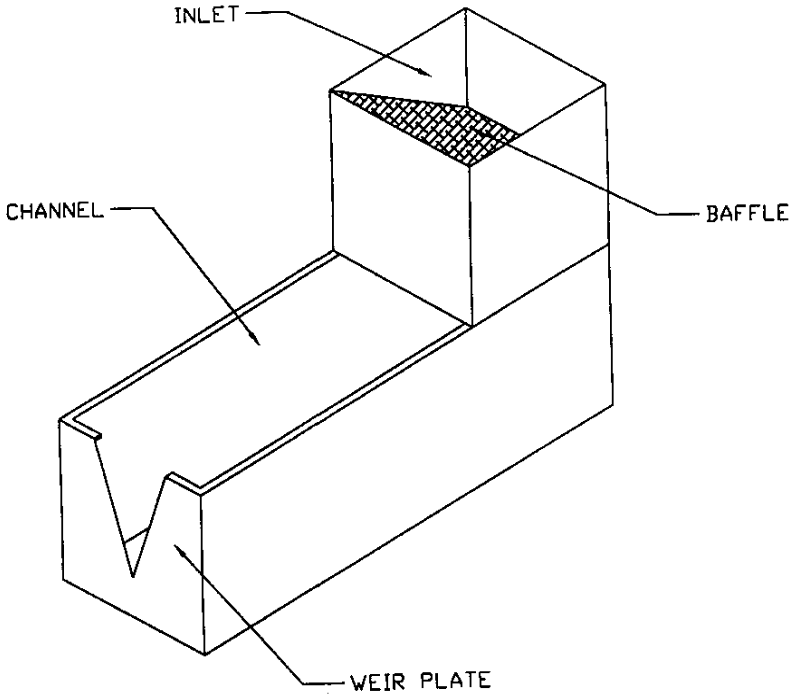

Figure 1: *The weir system measures flow rate as a function of the head measured above the weir crest. The water flow to be measured enters through the inlet and overflows through the weir plate. The baffles serve to lessen the turbulence of the water through the inlet.*

flow rates. From the observations, a weir coefficient will be determined." Bill moved up to the white board and wrote the following equation:

$$q = Kh^{5/2} \tag{1}$$

Bill explained, "This is the flow rate equation for thin plate weirs. q is the flow rate, K is the weir coefficient, and h is the head (depth of water above the weir crest). The weir coefficient is dependent on weir geometry. Therefore, once we figure out the coefficient, we will be able to use the equation to determine the flow rate during service." "Bill, what is the problem with a weir system that does not follow ASTM standards?" Laura asked. Laura Stephens was an engineer working on the development of doctor blade materials.

"I really don't remember, but it has to do with the channel effects on the flow. This is only important if you want to determine the weir coefficient following the ASTM equation for flow rate. Since we're going to use data from the calibration of the weir to determine the weir coefficient, we shouldn't have to worry about channel effects."

Al continued, "We have decided to use an ultrasonic depth sensor to measure the depth of the water in the channel during service. The function of the flow rate versus depth as found from the calibration will be used by the computer to determine the flow rate during service. Also, the computer interface will be able to output a flow rate to a chart recorder. With the chart recorder, we should be able to keep track of flow rates at the mill over an extended period with little maintenance." The ultrasonic device consisted of an ultrasonic sensor and a computer interface. The sensor is able to determine distance to a surface by measuring the time it takes for a signal it sends to bounce back from the surface. The sensor is able to measure distance from its base within an 8 degree cone up to distances of 20 feet. Any other echoes the sensor receives from outside the cone region are treated as noise. The sensor sends the information to the interface which calculates the depth by subtracting the distance measured by "0" level distance. The "0" level distance is the distance from the sensor to the water level where it is just below the overflow section of the weir plate. Bill interrupted, "The ultrasonic sensor just came in by mail. I'll go ahead and install it over the weir channel so we can test it during the calibration."

"How are you going to test it if the sensor can't determine the flow rate yet?" Asked Laura.

"Oh, it's able to measure the depth, so I will be able to compare depth readings from the ultrasonic sensor with actual mechanical measurements. Just to make sure the sensor is working properly."

Al concluded the meeting by stating that the project was almost complete. What was left was determining the real world effectiveness of the new doctor system. If the water removal rate is proven to be superior enough over the old system, then the doctor system may become a new product. "It's great that the mill is letting us try this unit on one of their machines, but we need to make sure we can obtain accurate data on the water removal. The paper machine is scheduled for shut down next month, and they will be ready to install the new doctor system then. We need to get the weir system up there soon to start taking data on the existing doctor system."

Calibration of the Weir System

Bill brought the weir system with the installed ultrasonic sensor over to the civil hydrology laboratory. Chris Bowman, a grad student working at the laboratory, installed the weir system in the laboratory. Water for the calibration was obtained from the main laboratory pond that had a gross gravity head (total height above the weir) of 18 feet. Chris was able to also use two centrifugal pumps to obtain higher flows.

"The flow rate into the weir system will be measured by a 2.5 inch turbine flow meter," Chris mentioned to Bill. "It's already been calibrated, and the flow meter does have an uncertainty of zero point five percent."

"All right, sounds good."

"Now how do you want this test to run?"

Bill replied, "It doesn't matter, I just need around 25 to 30 data points spread across the full flow rate range of the weir box. For each of the flow rates, we'll record the ultrasonic depth measurement along with the point gauge depth measurement."

The point gauge is a mechanical depth measurement device consisting of a caliper with a point on the bottom. The user moves the caliper up or down until the point on the bottom end just touches the water surface. The user can then measure the depth (or head) with good accuracy as long as the surface of the water measured is relatively calm.

Chris started the calibration by determining the zero level of water in the weir box. He sent a flow of water through until it started to spill over the weir notch. Chris turned off the water and measured the level of water in the channel after it appeared to be no longer spilling over the edge of the weir crest. This level was used to determine the head for the point gauge and the ultrasonic sensor. The head is the distance of the water level above the weir crest. To reference the ultrasonic sensor, Bill set it to "0" level.

Chris started a low flow rate through the weir system. He let the system come to equilibrium after a few seconds and then recorded the flow rate and the resulting head measurements made by the point gauge and the ultrasonic sensor. Chris then increased the flow rate and repeated the measurements. He continued over successive flow rate increments of around 5 to 15 GPM (gallons per minute). By the time the flow rate reached 140 GPM, the water surface in the weir channel was getting very wavy.

Chris gestured to Bill, "It's getting hard to measure with the point gage. Its going to be pretty hard to obtain an accurate reading. Bill, I could rig a stilling well around the point gauge. The stilling well will calm the surface of the water inside the well so the head measurement will be more accurate."

"Go ahead if it will help."

Chris asked his assistant to go look for one. The assistant came back with a clear plastic cylinder of around 8 inches in diameter and a foot and a half long. Chris clamped the cylinder on the side of the weir channel underneath the point gage. The cylinder set vertically with at least a foot clearance from the bottom of the weir channel. The surface of the water in the cylinder was much calmer.

"I would suggest that you install a permanent stilling well for that ultrasonic sensor as well. It should make the sensor more accurate at the higher flows. You can probably install a cylinder well like this one." Chris mentioned to Bill.

"Yeah, I see."

Chris finished taking measurements at higher flows. He continued to increase the flow rate till the water was just below the top of the weir plate. Noticing that he had only 25 measurements, he then added four more measurements at lesser flows.

Data Analysis

The weir system was brought back to Web Technologies, and Bill obtained the data from the calibration shown in Table 1. The weir coefficients were determined from equation (1) for both the ultrasonic sensor and the point gauge measurements by

$$K = q/h^{5/2} \tag{2}$$

K is the weir coefficient, q is the flow rate, and h is the head. Bill made a scatter plot of the weir coefficients versus flow of both the point gauge and the ultrasonic sensor (Figure 2).

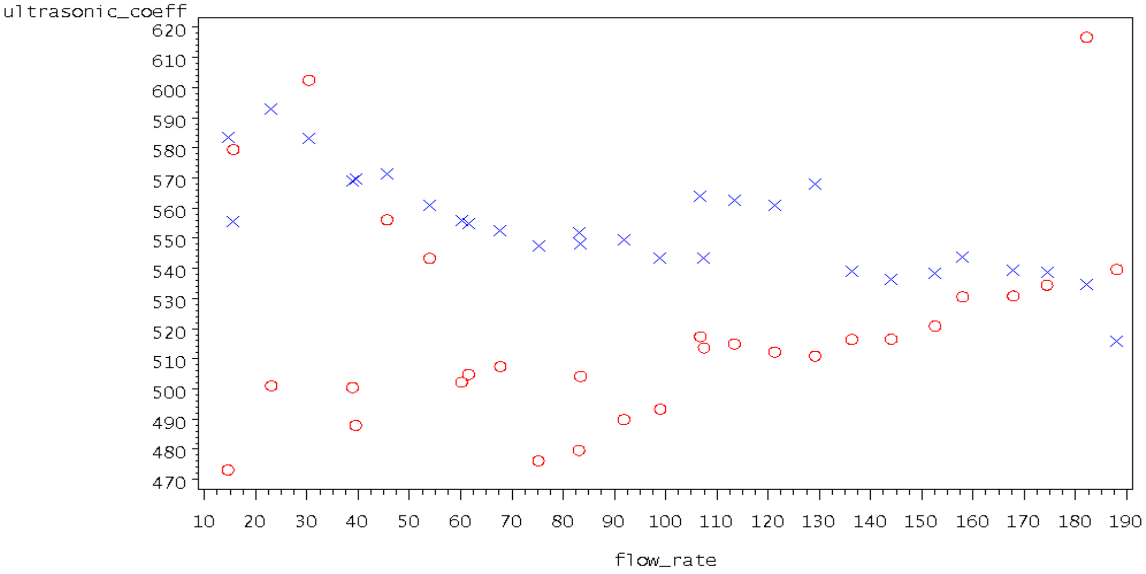

Figure 2: *Scatter plot of weir coefficients from ultrasonic sensor and the point gauge versus flow rate.*

Bill noted that the coefficients computed from the ultrasonic sensor measurements were more widely scattered across the flow rate range than those computed from the point gauge measurements. The scatter was most pronounced at the lower flow rates. Also, the coefficients determined with the ultrasonic sensor were mostly lower than the point gauge coefficients. Bill concluded that the ultrasonic sensor must be getting some interference at the low flow rates. The interference probably caused the wide scatter of data and inconsistency over the lower flow rates. It was also possible that the "0" level measured by the ultrasonic sensor could have some interference that might throw off the measurements. Such an incorrect "0" level would explain the consistently low ultrasonically-computed coefficients.

Next, Bill looked at the effect of the stilling well had on the weir coefficients determined by the point gauge. He decided to compare the coefficients computed from the data obtained without the stilling well to those computed from the data obtained with the stilling well. To make a comparison, Bill constructed the scatter plot of the data shown in Figure 3.

Based on Figure 3, Bill concluded that the stilling well did have a large effect on the coefficients above 100 GPM. Bill decided that a stilling well was needed for the ultrasonic sensor, so the weir coefficient

Run	Flow	Head		Weir Coefficient	
		Ultrasonic	Gauge	Ultrasonic	Gauge
1	14.64	0.249	0.229	473.2	583.4
2	15.68	0.236	0.240	579.5	555.7
3	23.09	0.292	0.273	501.1	592.9
4	30.45	0.303	0.307	602.5	583.1
5	38.92	0.360	0.342	500.5	569.0
6	45.69	0.368	0.364	556.2	571.6
7	53.96	0.397	0.392	543.4	560.9
8	61.58	0.431	0.415	504.9	555.0
9	67.79	0.447	0.432	507.5	552.7
10	75.22	0.478	0.452	476.2	547.6
11	83.12	0.496	0.469	479.7	551.8
12	91.89	0.512	0.489	489.9	549.5
13	99.00	0.526	0.506	493.4	543.6
14	106.81	0.532	0.514	517.4	563.9
15	113.45	0.546	0.527	515.0	562.7
16	121.31	0.562	0.542	512.3	560.9
17	129.22	0.577	0.553	511.0	568.2
18	136.36	0.587	0.577	516.5	539.2
19	144.07	0.600	0.591	516.6	536.5
20	152.64	0.612	0.604	520.9	538.4
21	158.03	0.616	0.610	530.6	543.8
22	167.91	0.631	0.627	530.9	539.4
23	174.46	0.639	0.637	534.5	538.7
24	182.22	0.614	0.650	616.8	534.9
25	188.11	0.656	0.668	539.7	515.8
26	107.54	0.535	0.523	513.7	543.6
27	83.45	0.487	0.471	504.2	548.1
28	60.20	0.428	0.411	502.3	555.9
29	39.55	0.366	0.344	488.0	569.8

Table 1: *Data collected from the calibration of the weir system. Flow is in gpm, head in ft.*

to be used in service should be calculated with data obtained with a stilling well. However, this did not leave Bill with sufficient data to quantify the relation of the weir coefficients to the flow rate. He decided he would need to obtain more data on the flow rate and corresponding depth.

Bill went over to Al's office to update him on the weir system progress.

"So how did the calibration go, Bill?" Al asked.

"It went as well as possible. I have the calibration report along with some plots I made right here." Bill proceeded to show the scatter plots of the results he had obtained. "The one thing I did learn is that we need a stilling well in the weir channel. The surface of the water got pretty rough at the higher flows. Also, there appears to be some interference with the ultrasonic sensor at the lower flow rates. I think that the sensor is too close to the channel walls. I will have James reinstall the sensor with a stilling well. Hopefully we can mount the sensor with no obtrusions in its echo range." James Parr was a summer intern working in the research and development department.

Al replied, "Sure, I think it will be a good project for James. Now do we have a coefficient to use to determine flow rate?"

"Just looking at the data obtained with stilling well, it looks like the coefficient is a function of the flow rate. Just using the average coefficient would not work well. I can find the equation of the best fit curve for the coefficient. We can enter an equation of the coefficient as a function of depth into the computer."

Al asked, "Shouldn't the coefficient be constant?"

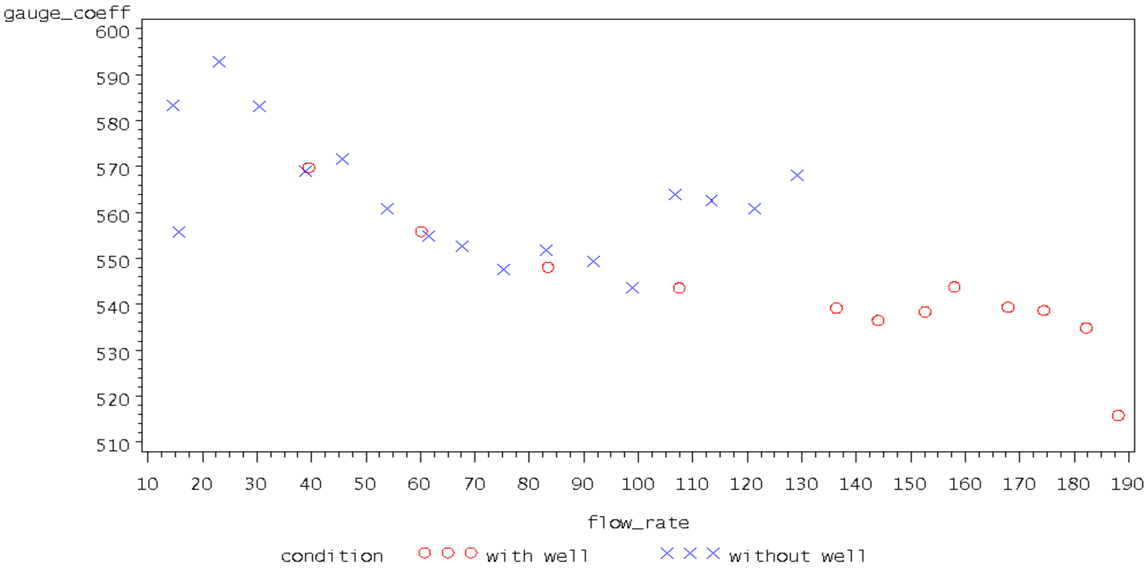

Figure 3: *Scatter plot of the point gauge weir coefficients versus flow rate. 'with well' were measured with a stilling well in place. 'without well' are measurements taken without the stilling well.*

"It should be for a weir system made to ASTM standards. But, our weir isn't. It looks like channel effects are causing variation in the coefficients."

"So where do we stand now?"

"Well, I am afraid I will need to re-calibrate the weir system. After James remounts the ultrasonic sensor with the stilling well, I will take the system back to the hydrology laboratory. We'll take measurements and make sure the ultrasonic sensor is working correctly."

"Okay, but I would like to have the system up in the mill and running within a week."

Remounting the Ultrasonic Sensor

Bill gathered the weir data and went to inform James about his new project. Bill gave a copy of the data and lab report to James.

Bill and James studied the position of the sensor over the weir channel. They noticed that the sensor's stand positioned the sensor close to the weir inlet box. They concluded that the stand was within the 8 degree cone below 0.4 feet above the weir notch. This coincides with a flow of around 50 GPM. From Figure 2, the ultrasonic sensor appeared to be most erratic below 50 GPM. Bill told James to increase the span of the support for the sensor to position the sensor out of range of the stand or the weir channel side.

James made a new support for the ultrasonic sensor with a longer span. Also, he had a stilling well mounted in the channel underneath the sensor. James paid careful attention to make sure nothing (including the stilling well) would interfere with the sensor's measurement down to the "0" level of the weir system.

James decided to test the sensor's accuracy over the range of water head found in the weir. To test the sensor, he placed a large bucket in the channel under the sensor and stilling well. He filled the bucket with water until the level was close to the actual "0" level of the weir system. A reference "0" level was made for both the sensor and the digital caliper. James then started to take head measurements with the ultrasonic sensor and with the digital caliper. By adding water to the bucket, measurements of depth were obtained from both the ultrasonic sensor and the caliper. James continued taking data at various levels until the depth matching the top of the of the weir channel was reached. Table 2 shows the results that James obtained.

The descriptive statistics of the residuals are as follows:

Run	Ultrasonic	Caliper	Residual
1	0.254	0.254	0.000
2	0.281	0.279	−0.002
3	0.296	0.298	0.002
4	0.318	0.315	−0.003
5	0.327	0.329	0.002
6	0.344	0.343	−0.001
7	0.36	0.360	0.000
8	0.375	0.373	−0.002
9	0.384	0.384	0.000
10	0.397	0.398	0.001
11	0.408	0.408	0.000
12	0.42	0.417	−0.003
13	0.428	0.430	0.002
14	0.441	0.441	0.000
15	0.457	0.455	−0.002
16	0.464	0.467	0.003
17	0.478	0.479	0.001
18	0.495	0.494	−0.001
19	0.509	0.510	0.001
20	0.524	0.524	0.000
21	0.539	0.538	0.00
22	0.552	0.550	−0.002
23	0.561	0.562	0.001
24	0.582	0.582	0.000
25	0.601	0.599	−0.002
26	0.613	0.612	−0.001
27	0.627	0.627	0.000
28	0.64	0.643	0.003
29	0.659	0.657	−0.002
30	0.672	0.671	−0.001
31	0.684	0.685	0.001
32	0.696	0.697	0.001
33	0.682	0.681	−0.001
34	0.665	0.665	0.000
35	0.647	0.646	−0.001
36	0.625	0.627	0.002

Table 2: *Analysis of the ultrasonic sensor. The residual is the difference between the caliper measurement and the ultrasonic measurement.*

N	36
Mean	−0.0001
Std Dev	0.0016
Minimum	−0.003
1st Quartile	−0.001
Median	0
3rd Quartile	0.001
Maximum	0.003

A scatter plot of the residuals versus the ultrasonic measurement is shown in Figure 4. The residuals do appear to be random; therefore, James concluded that the error in the ultrasonic sensor should be stable over the entire range of head measurement.

James wanted to obtain an idea of how accurate the ultrasonic measurements were as compared to the measurements taken by the caliper. James first decided to obtain a confidence interval for the mean of the errors. If the interval contained zero, then James would have little reason to believe the ultrasonic measurements were biased. Bias is a statistical term that means that the mean value of an estimator differs from the quantity being estimated. In this application, the ultrasonic sensor is biased if it measures too high or too low on average: that is, if the mean of the residuals is non zero. A 95% confidence interval for the mean of the residuals is given by the formula:

$$\overline{Y} \pm (S/\sqrt{n})t_{35,0.975}$$

where $\overline{Y}$ is the sample mean, S is the standard deviation, and n is the number of observations. James determined the quantile $t_{35,0.975}$ was equal to 2.0301 from a table of critical values of the t distribution. James then determined the confidence interval:

$$[-0.0001 - (0.0016/6)(2.301), -0.0001 + (0.0016/6)(2.301)] = (-0.000716, 0.000514)$$

Since the confidence interval for the mean contains zero, James felt confident in assuming that the ultrasonic measurements were not biased.

James then decided to determine a tolerance interval for the error of the ultrasonic sensor to assess how close the measurements are to the actual measurement. James assumed that the caliper measurements

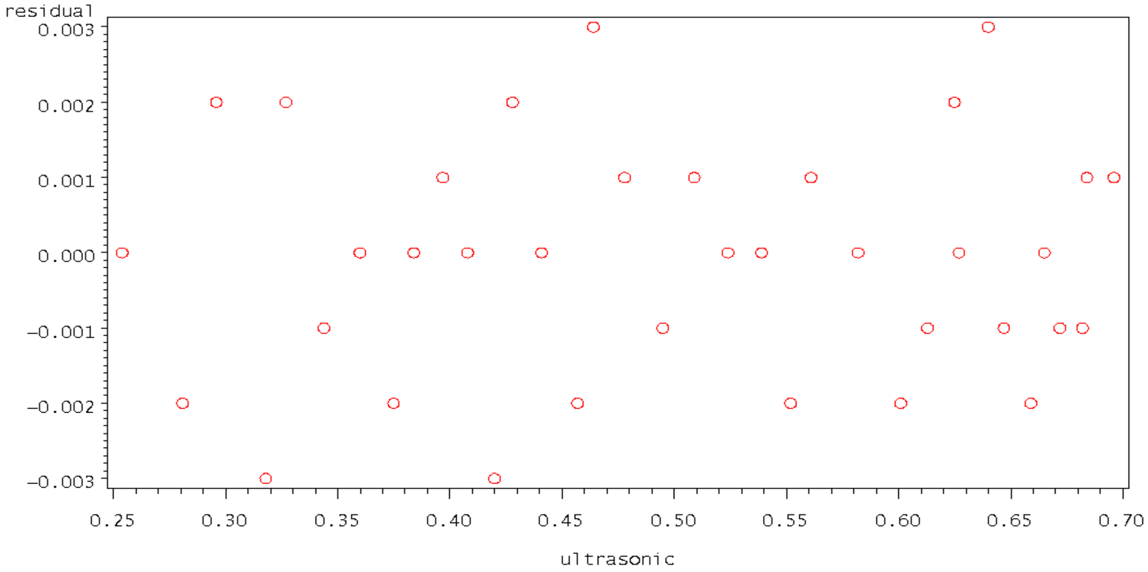

Figure 4: *Difference between the ultrasonic and caliper measurements plotted against the ultrasonic depth measurement.*

were the true values. A tolerance interval is used to give a range that will contain at least a proportion of the population with a pre-specified confidence. The level L tolerance interval for a proportion γ of the population is given by:

$$\overline{Y} \pm KS,$$

where $\overline{Y}$ is the sample mean, S is the sample standard deviation, and K is a mathematically derived constant. James decided to find a 95% tolerance interval ($L = 0.95$) for 99% proportion of the population ($\gamma = 0.99$). From a table of constants for Normal-Theory Tolerance Intervals, K was found to be 3.272. James then calculated the tolerance interval:

$$[(-0.0001 - (3.272)(0.0016)), (-0.0001 + (3.272)(0.0016))] = (-0.0053, 0.0051)$$

So James was 95% confident that at least 99% of all errors obtainable from the ultrasonic sensor lay between -0.0053 and 0.0051. 95% confidence means if samples of error measurements are repeatedly taken and for each set of measurements a 95% level tolerance interval for a 99% proportion of the population is computed, then 95% of all those intervals will actually contain 99% or more of the population of all error measurements.

However, the tolerance interval was determined from data obtained under ideal conditions since there was no turbulence on the water surface during measurements. The interval is not good for actual use because there is going to be some turbulence on the surface of the water despite the stilling well. The interval does show that the ultrasonic sensor works well in its new location and under ideal conditions would obtain measurements very close to measurements taken mechanically.

Re-calibration of the Weir System

After Bill was updated by James on the success of mounting the ultrasonic sensor, Bill made plans to take the weir system back to the hydrology laboratories. At the laboratory, the same procedure as the first calibration was followed. Chris set up the test and 24 different flow rates over the flow rate range of the weir. At each of the flows the measurement of the head was made with both the ultrasonic sensor and the point gauge. Table 3 shows the results taken from the second calibration.

Back at his office, Bill plotted the results of the weir coefficients from both the ultrasonic sensor and the point gauge (Figure 5).

Run	Flow	Head		Weir Coefficient	
		Ultrasonic	Gauge	Ultrasonic	Gauge
1	15.55	0.232	0.231	599.8	606.3
2	16.54	0.238	0.238	598.5	598.5
3	21.05	0.258	0.259	622.6	595.6
4	31.33	0.312	0.311	576.2	576.8
5	41.58	0.351	0.349	569.7	567.1
6	46.10	0.363	0.365	580.7	564.0
7	52.87	0.388	0.388	563.8	557.5
8	64.68	0.424	0.423	552.5	555.8
9	73.77	0.449	0.448	546.1	551.9
10	83.57	0.47	0.471	551.8	548.9
11	91.29	0.492	0.492	537.7	547.7
12	98.52	0.508	0.505	535.6	543.6
13	101.67	0.513	0.513	539.4	544.3
14	108.72	0.524	0.525	547.0	544.4
15	114.94	0.536	0.538	546.5	541.4
16	120.36	0.549	0.549	539.0	544.0
17	134.00	0.574	0.573	536.8	539.2
18	137.80	0.583	0.58	531.0	541.9
19	148.84	0.598	0.599	538.2	536.0
20	156.35	0.611	0.611	535.8	540.8
21	165.30	0.624	0.626	537.4	540.1
22	171.49	0.634	0.633	535.8	537.9
23	182.60	0.651	0.652	534.0	539.0
24	189.69	0.663	0.67	530.0	516.2

Table 3: *Data from the second calibration of the weir system. Flow is in gpm, head in ft.*

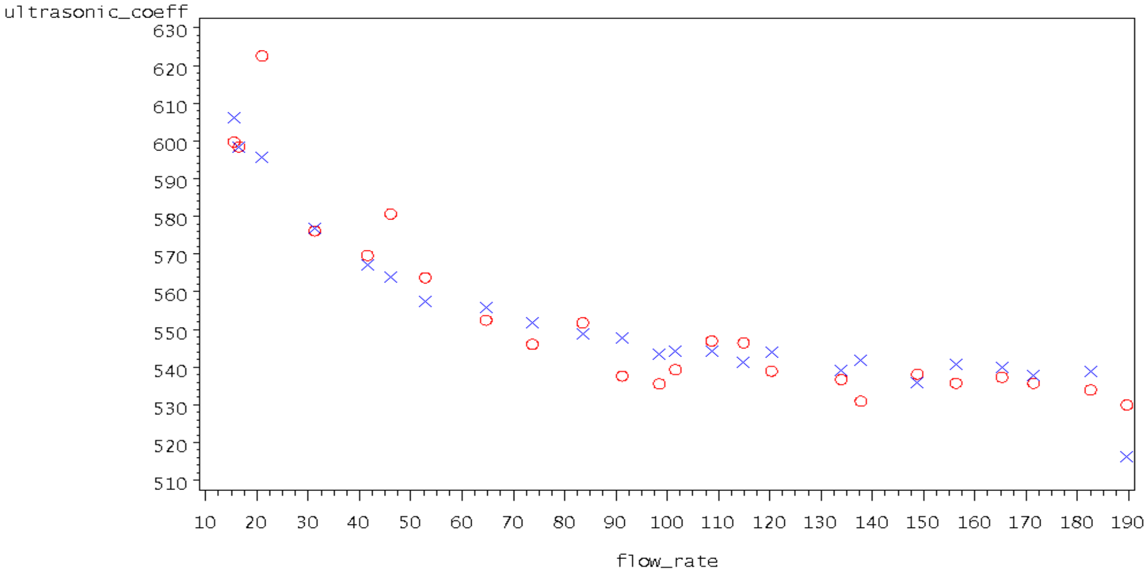

Figure 5: *Scatter plot of both the ultrasonic sensor and point gauge head versus flow rate.*

From the scatter plot, it was clear that the weir coefficients determined from the ultrasonic sensor data were relatively close to those obtained from the point gauge data. However, there is a variable relationship between the coefficients and the flow rate. Bill decided to see how the variability will effect the flow measurement. Bill determined the average coefficient from the coefficients determined by the point gauge. He plugged the average coefficient in equation (1) obtaining:

$$\text{Flow Rate (GPM)} = 553.3[\text{Head}]^{5/2} \tag{3}$$

To determine how good the equation was, Bill made a residual plot shown in Figure 6. A residual plot is used to assess the quality of fit and adequacy of model assumptions. The residual is the difference between actual value and the fitted value. Bill plotted the residual plot with the residuals against the predicted flow rate. From the residual plot, it was clear to Bill that error in flow rate prediction was not stable across the flow rate range. Therefore, this model would not be a good relation to use in order to predict the flow rate.

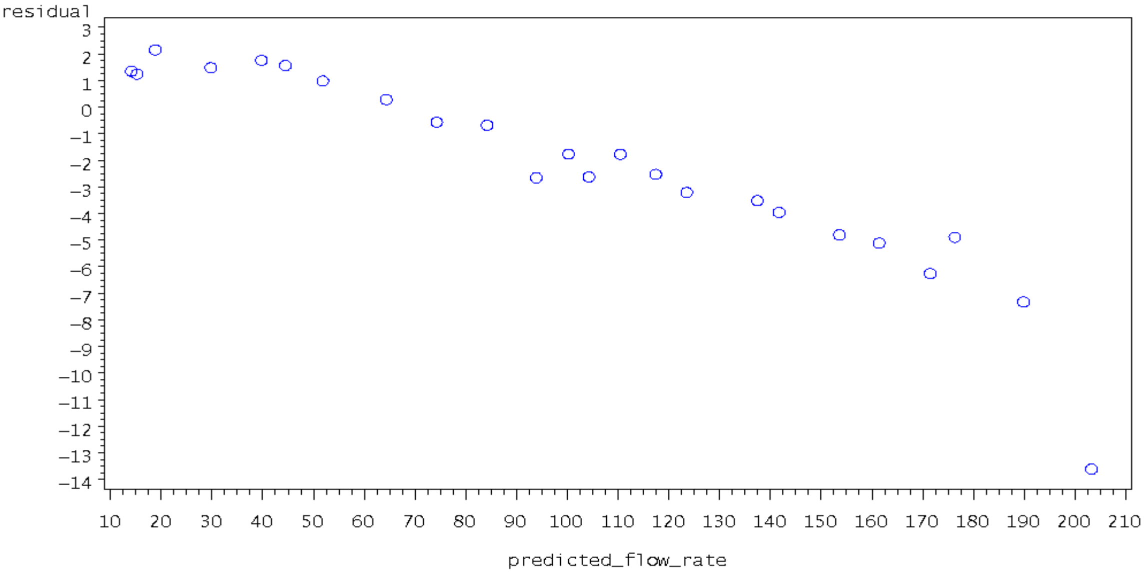

Figure 6: *Scatter plot of the difference between the fitted value using the average coefficient and the actual value plotted against the predicted flow rate value.*

Since using equation (1) with a constant coefficient would not work, Bill decided to determine the flow rate as a function of the head. First, Bill wanted to determine how the second calibration data from the point gauge compared with the original calibration data taken with the stilling well. Bill made a scatter plot of both the first calibration and second calibration coefficients made with the point gauge (Figure 7). From the scatter plot, it appeared that the data from both calibrations were consistent with each other. Bill decided to use both the original data taken with the stilling well and the second calibration data to determine the flow rate as a function of the head.

Bill made a scatter plot of the flow rate versus the head from data obtained from both calibrations with the stilling well (Figure 8).

In attempt to linearize the data, Bill then plotted the flow rate versus the head$^{(}$5/2) shown in Figure 9.

Bill took the depth to the (5/2) power because of equation (1). The data looked linear and Bill made regression of the data to obtain a function of the flow to the head:

$$\text{Flow Rate (GPM)} = 3.75 + 521.2\text{Head}^{5/2} \tag{4}$$

This fit gave an R^2 and an adjusted R^2 value of 0.9991, and $\sqrt{\text{MSE}}$ of 1.6889.

Bill made a residual plot to determine the quality of the fit of the regression (Figure 10).

A residual plot plots the difference between the actual flow rate and the predicted flow rate plotted against the predicted flow rate.

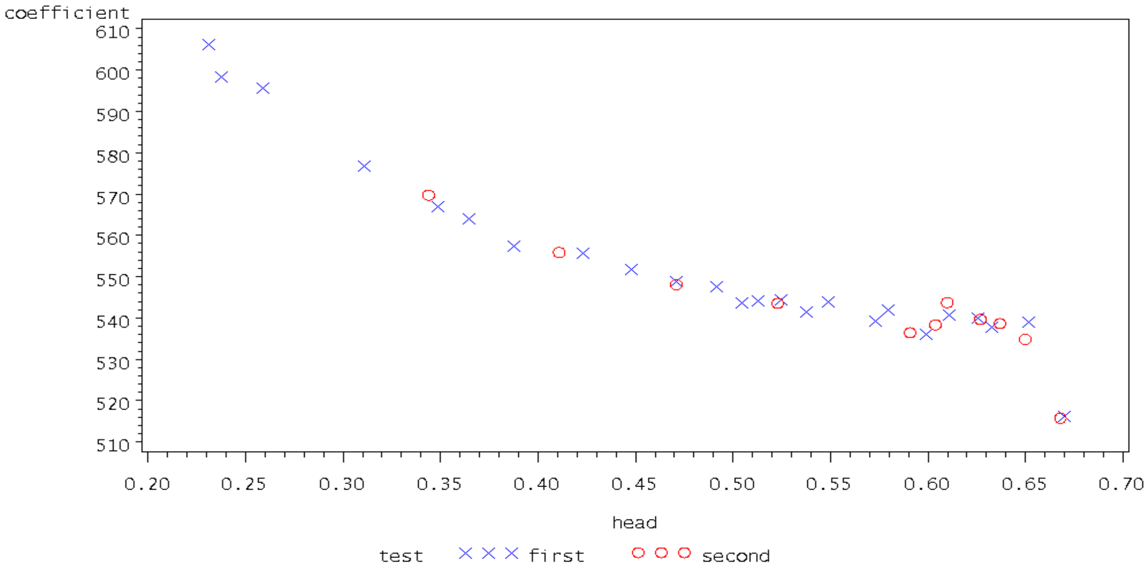

Figure 7: *Scatter plot of the weir coefficients determined from the first and second weir calibrations. The first test data were obtained with the stilling well in place around the point gauge.*

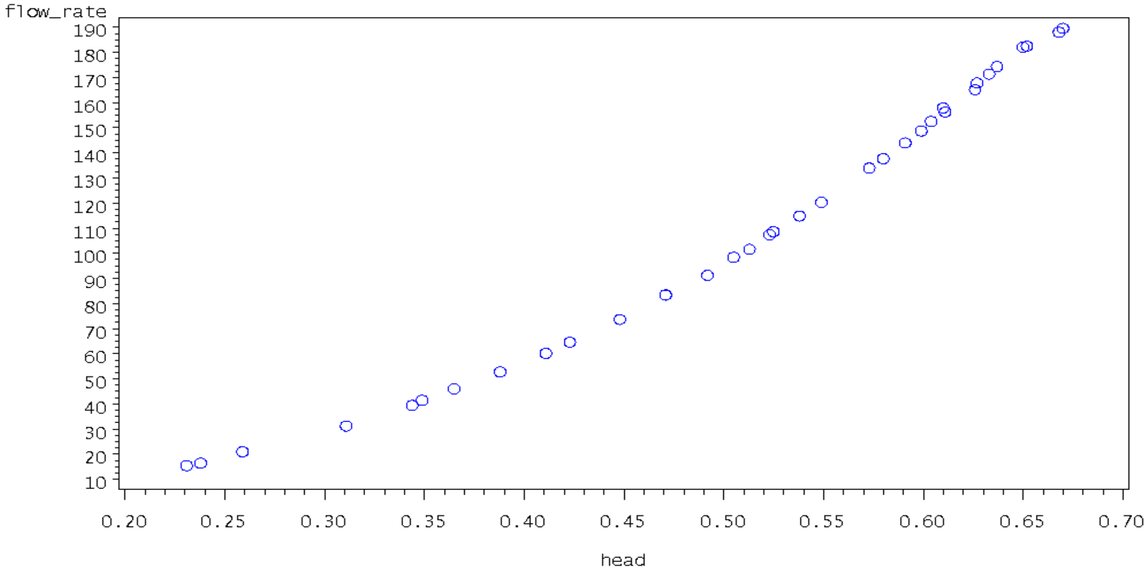

Figure 8: *Scatter plot of the flow rate versus head of the original calibration data made with the stilling well and the second calibration data.*

From the residual plot, Bill noted the two outliers found at the highest flow measurements. He remembered that Chris continued taken data measurements during the calibration till the depth of the water reached the top of the weir channel. Since the outliers were obtained at the highest flow rates, Bill decided that they were measured out of the flow range of the weir system. Flow rates over 185 GPM would not be seen in service. However, Bill noted in his lab book that the weir system should only be used to measure flow rates between 15 to 185 GPM. Bill decided he could probably obtain a better regression fit if these data points were taken out. With the outliers taken out, Bill obtained the following equation from a regression fit:

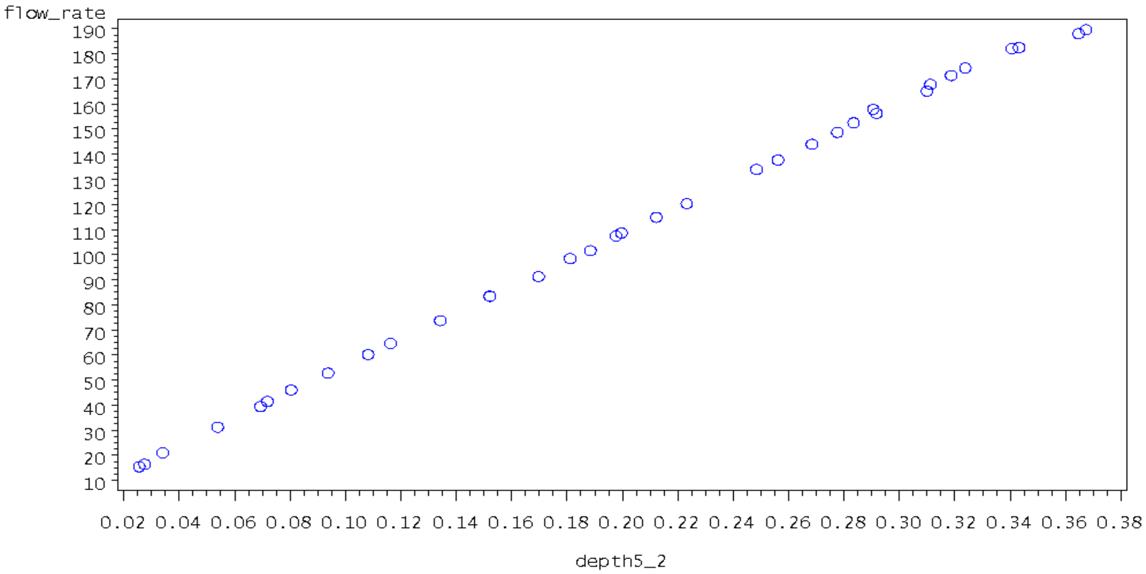

Figure 9: *Scatter plot of the flow rate versus depth to the 5/2 power.*

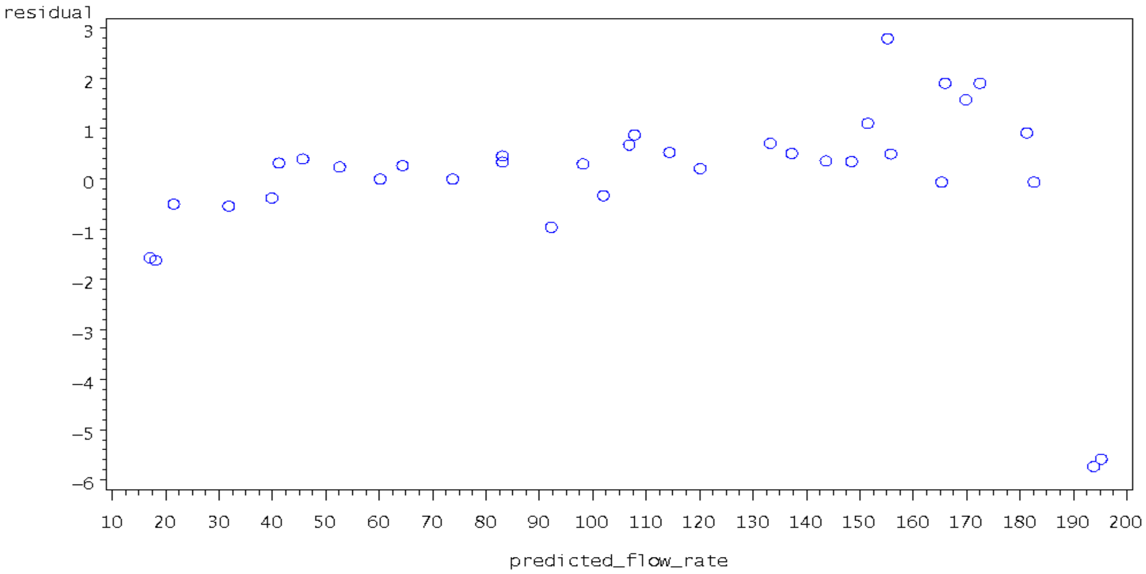

Figure 10: *Residual plot of the difference between the actual flow rate and the predicted flow rate plotted against the predicted flow rate.*

$$\text{Flow Rate (GPM)} = 2.94 + 527.2\text{Head}^{5/2} \tag{5}$$

This fit gave an R^2 and an adjusted R^2 value of 0.9998, and $\sqrt{\text{MSE}}$ of 0.6829.

A plot of the residuals versus the predicted flow rate with the two outliers removed is shown in Figure 11. The residuals are the difference of the flow rate from the calibration and the predicted flow rate.

Bill determined from the residuals that the error in flow prediction is relatively constant over the flow rate range. Besides the 2.94 offset, the ASTM flow rate equation (1) appears to work with a weir coefficient of 527.2. Bill decided to use equation (5) for determining the actual flow rate during service.

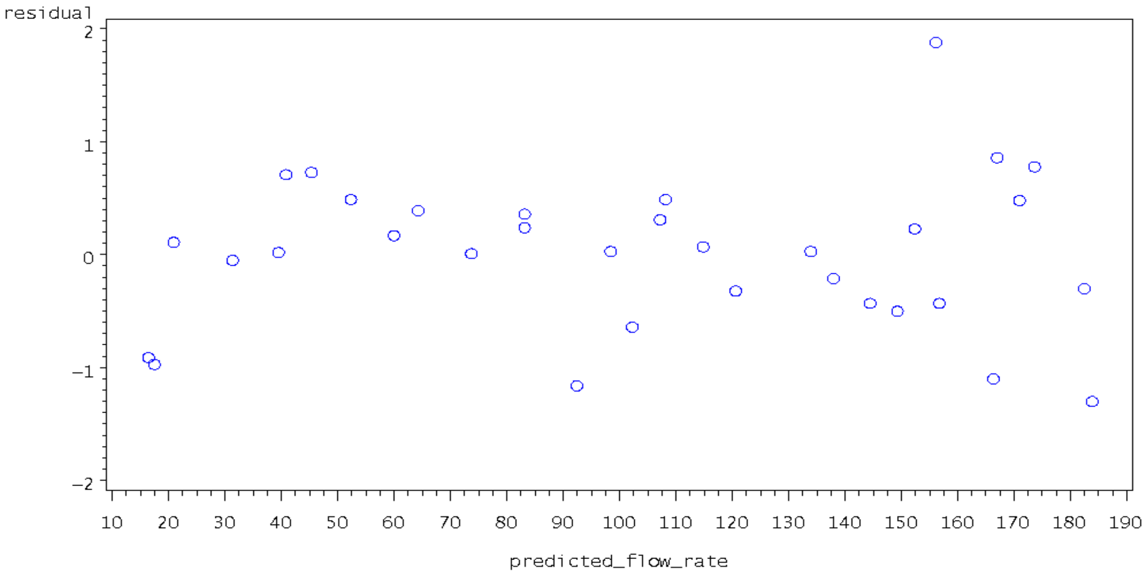

Figure 11: ***Residual plot of the difference between the actual flow rate and the predicted flow rate plotted against the predicted flow rate.***

Bill entered the flow rate equation in the computer interface for the ultrasonic sensor. He also made preparations for the system to be sent out to the mill. The weir system was ready just in time to get it up to the mill to take flow rate measurements on the old doctor system. Bill went to see Al Showman to tell him about the progress that had been made.

"So everything is ready to go?" Al asked.

"Yes, the weir system is being crated and should be at the mill by tomorrow evening." Bill replied.

"With the second calibration, were you able to obtain a good relation of the flow rate to the measured head?"

Bill answered yes and showed Al the statistical analysis of the calibration data. Bill explained, "Turns out that the ASTM equation for measuring flow rate works well for our weir system. Originally, when I was just looking at the weir coefficients determined from each observation, I thought obtaining an equation was going to take a bit of work. I would have to find the relation of the weir coefficient to the depth and then plug that back into the ASTM flow rate and it would have been ugly. However, once I took a step back and just concentrated on finding a relation of the flow rate to the depth, it all simplified nicely. I just couldn't figure out what caused that small offset." Bill pointed to the 2.94 offset in equation (5).