Chapter 1: Introduction to Data Analysis

Preview:

- Data and its science, statistics
- Stationary and nonstationary processes; displaying data from each.
- Assessing between and within variation.

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What’s the IDEA?

- Data have variation.
- The variation has a pattern (data distribution).
- By analyzing the pattern, we can tell something about the process or population the data came from.

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What are data?

Data are facts that convey information. One example is the GLOBAL_TEMPS data set which you imported into SAS Studio in lab 1. This data set contains the average global surface air temperature anomalies in degrees celsius for the years 1880-2016. An anomaly is the difference between the average temperature for a given period and a baseline average. This data set contains monthly and yearly anomalies compared with a baseline computed from the years 1951-1980. (Source: http://data.giss.nasa.gov/gistemp/)

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Here is a portion of the data set:

Year	anomaly_c	anomaly_f	temp_c	temp_f
1880	− 0.43	−0.774	13.97	57.146
1881	− 0.36	−0.648	14.04	57.272
1882	− 0.31	−0.558	14.09	57.362
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
2014	0.87	1.566	15.27	59.486
2015	0.97	1.746	15.37	59.666
2016	1.23	2.214	15.63	60.134

Displaying a Static Pattern of Variation

A Frequency Histogram shows a **Data Distribution**: i.e., static pattern of variation. A histogram of the yearly temperature anomalies from the global temperature data set is shown in Figure 1.

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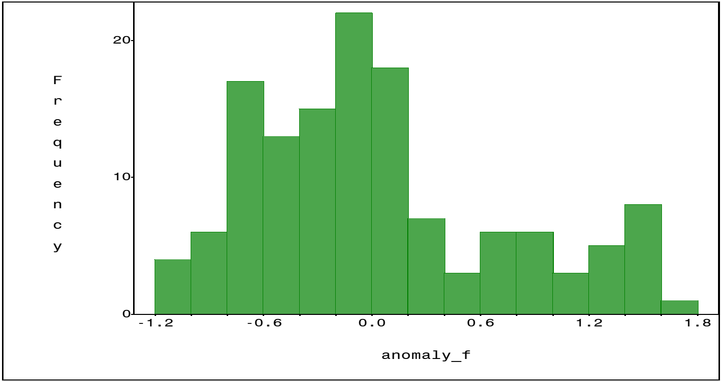

Figure: Figure 1: Histogram of anomalies, degrees fahrenheit, global temperature data.

Can this histogram tell us anything about the possible existence of global warming?

Displaying Data Taken Over Time

A **Time Series Plot (or Line Plot)**: shows pattern of variation evolving over time. Figure 2 displays a time series plot of the temperature anomalies in fahrenheit from the global temperature data set (along with the previous histogram turned on its side).

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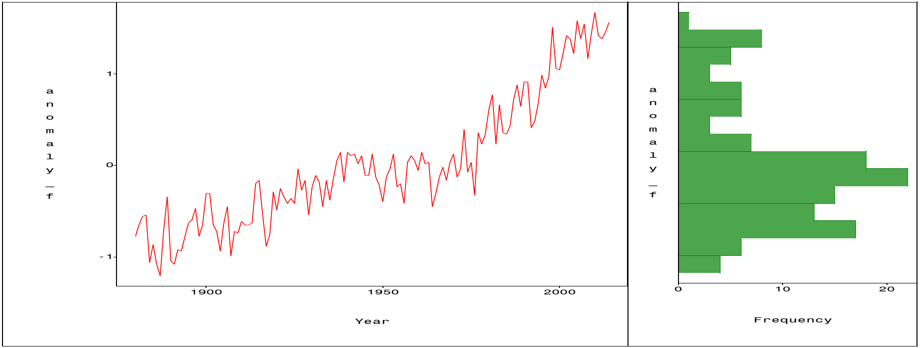

Figure: *Figure 2: Time series plot of anomalies, degrees fahrenheit (left); histogram (right), global temperature data.*

Can it tell us anything about the possible existence of global warming?

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Displaying Data Taken Over Time

Figure 3 is a fake time series plot of the anomalies we have already plotted in Figures 1 and 2. It was formed by reordering the time sequence.

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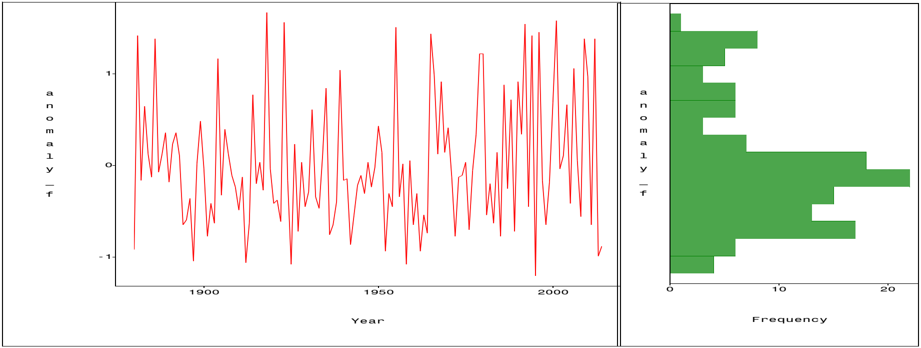

Figure: *Figure 3: A fake time series plot of anomalies, degrees fahrenheit (left); histogram (right), global temperature data.*

This plot has the same histogram shown in Figures 1 and 2. What does this say about choosing the type of plot for displaying data?

Displaying Data Taken Over Time

What's the IDEA?

The first step in analyzing data should **ALWAYS** be to plot it.
BUT... be sure to use appropriate plots.

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Stationary Processes

- A process is **stationary** if the pattern of variation does not change as more data are taken.
- **To assess stationarity** data must be plotted versus time.

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Stationary Processes

Figure 4 shows side-by-side the real and fake time series plots of the anomaly data. Which, if either, is stationary?

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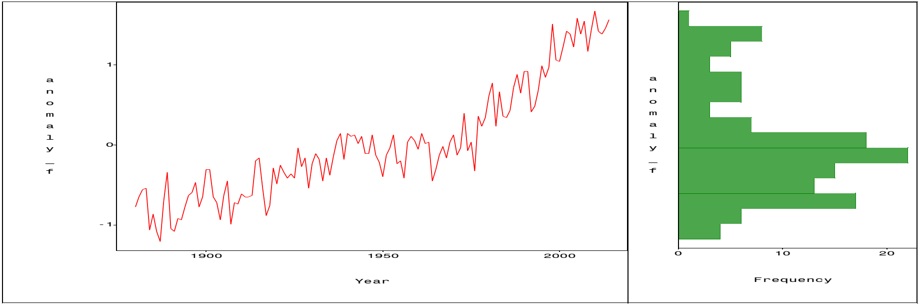
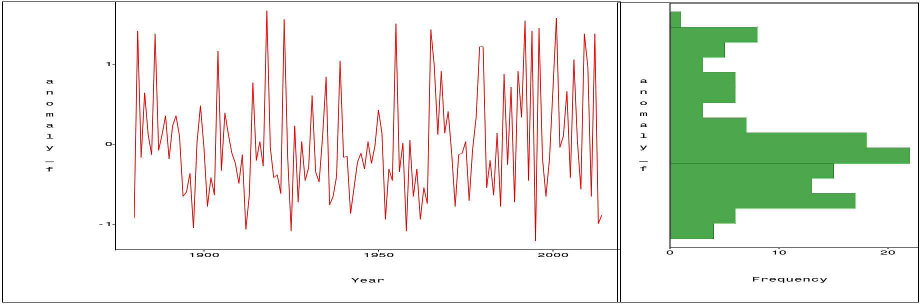

anomaly_f
1
0
-1
1900
1950
2000
Year
anomaly_f
0
10
20
Frequency


anomaly_f
1
0
-1
1900
1950
2000
Year
anomaly_f
0
10
20
Frequency

Stationary Processes

The implication for graphical data displays of data taken over time, is that if the data are stationary, then plots displaying static patterns of variation are ok.

For this example, the histogram tells the story of the temperature anomalies if the data are stationary, as in the fake time series plot, but does not if the data are not stationary, as in the real time series plot.

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What’s the IDEA?

If data are taken over time, **ALWAYS** check stationarity by plotting versus time. If the pattern is nonstationary, displays and measures designed for data from a stationary process are probably inappropriate.

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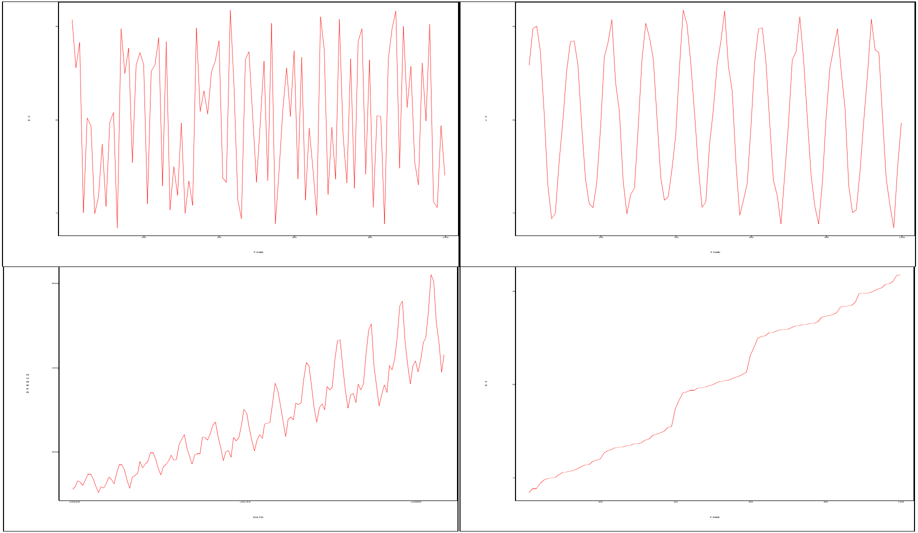
Are these variation patterns stationary or nonstationary? Why?



Comparing Two or More Data Sets

Often, we want to compare two or more components of a process, as in the following example.

A bakery finds too much variation in the weights of its 1 lb loaves of white bread. This bread is produced on 3 machines. Which machine(s) is responsible for the problem?

To find out, 8 loaves are randomly sampled from each machine's production on a given day and weighed. For each machine, the weights are plotted versus the time order of production and no evidence of nonstationarity is found.

A plot appropriate for comparing data from stationary processes is a **stratified plot**. The stratified plot for the bakery data is shown in Figure 5.

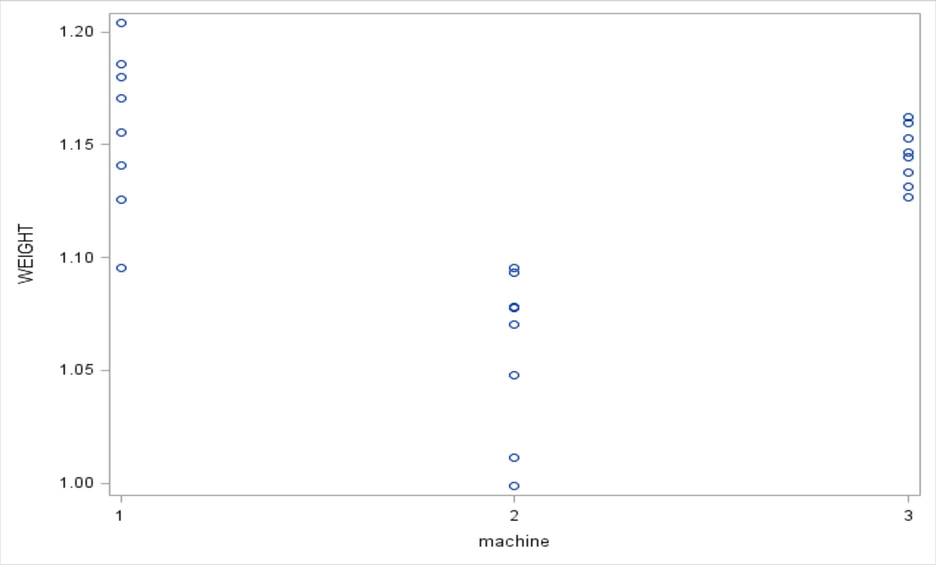
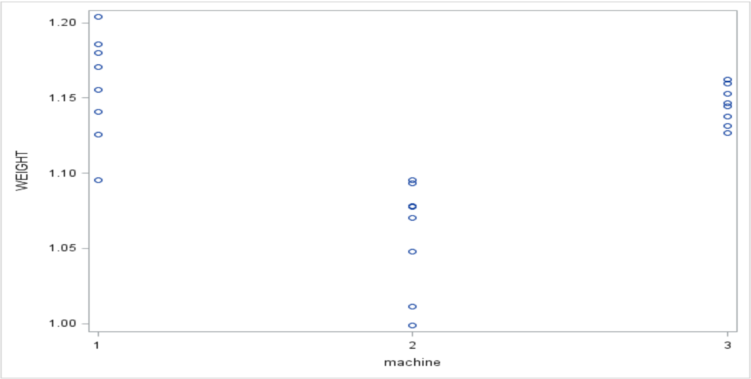


Figure: *Figure 5: A stratified plot of the weights of bread from 3 machines.*

The plot is called stratified because the data are broken into groups called **strata** and the distributions for the different strata are compared. The strata in this example are the machines.

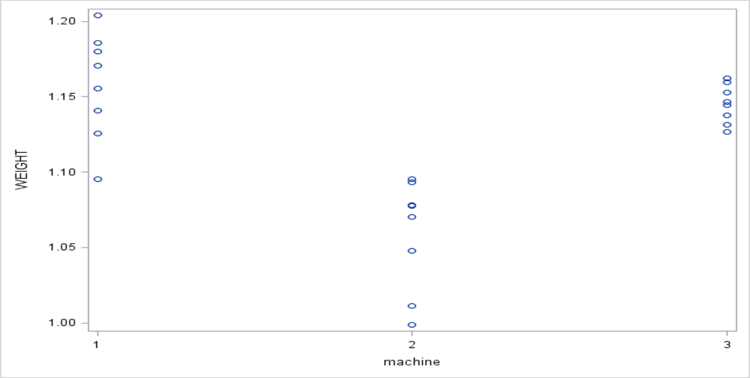
The things to look for in a stratified plot are the variation or spread within strata (**within variation**) and the differences in the locations or centers of the data in the different strata (**between variation**).

Have another look at the stratified plot of the bread weights and compare the within and between variation.



The plot seems to show substantial between and within variation.

The principal within variation is in machines 1 and 2: the variation in the weights produced by both machines is about the same and machine 3 produces bread with more consistent weights than either.



The between variation derives from the central value of the weights of bread produced by machine 2 being less than the central values in the weights produced by either machine 1 or 3. The central values for machines 1 and 3 are about equal.

But how much of the variation is the result of differences in machines?

Measuring the Variation

We can measure the between, within, and total variation in the bakery data as follows. First, look at the data.

	Machine		
Loaf	1	2	3
1	1.1799	0.9987	1.1269
2	1.1409	1.0783	1.1595
3	1.1256	1.0778	1.1442
4	1.2039	1.0706	1.1620
5	1.1703	1.0956	1.1378
6	1.1858	1.0113	1.1269
7	1.1553	1.0476	1.1316
8	1.0952	1.0935	1.1466
Mean	1.1571	1.0592	1.1452

The overall mean is 1.1205

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To measure the **total variation**, subtract the overall mean from each loaf's weight, then square the results and add up all those squares:

	Machine		
Loaf	1	2	3
1	$(1.1799 - 1.1205)^2$	$(0.9987 - 1.1205)^2$	$(1.1269 - 1.1205)^2$
2	$(1.1409 - 1.1205)^2$	$(1.0783 - 1.1205)^2$	$(1.1595 - 1.1205)^2$
3	$(1.1256 - 1.1205)^2$	$(1.0778 - 1.1205)^2$	$(1.1442 - 1.1205)^2$
4	$(1.2039 - 1.1205)^2$	$(1.0706 - 1.1205)^2$	$(1.1620 - 1.1205)^2$
5	$(1.1703 - 1.1205)^2$	$(1.0956 - 1.1205)^2$	$(1.1378 - 1.1205)^2$
6	$(1.1858 - 1.1205)^2$	$(1.0113 - 1.1205)^2$	$(1.1269 - 1.1205)^2$
7	$(1.1553 - 1.1205)^2$	$(1.0476 - 1.1205)^2$	$(1.1316 - 1.1205)^2$
8	$(1.0952 - 1.1205)^2$	$(1.0935 - 1.1205)^2$	$(1.1466 - 1.1205)^2$

The resulting value is 0.0650. This is called the **total sum of squares**.

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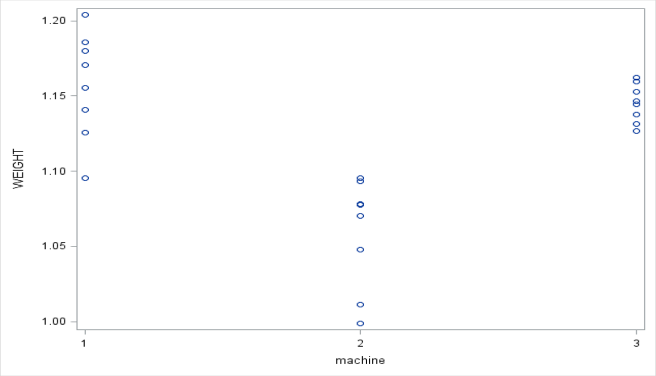
To measure the **within variation**, first subtract from each loaf's weight the mean weight for the loaves produced by the machine that manufactured it, then square the results and add up all those squares:

	Machine		
Loaf	1	2	3
1	$(1.1799 - 1.1571)^2$	$(0.9987 - 1.0592)^2$	$(1.1269 - 1.1452)^2$
2	$(1.1409 - 1.1571)^2$	$(1.0783 - 1.0592)^2$	$(1.1595 - 1.1452)^2$
3	$(1.1256 - 1.1571)^2$	$(1.0778 - 1.0592)^2$	$(1.1442 - 1.1452)^2$
4	$(1.2039 - 1.1571)^2$	$(1.0706 - 1.0592)^2$	$(1.1620 - 1.1452)^2$
5	$(1.1703 - 1.1571)^2$	$(1.0956 - 1.0592)^2$	$(1.1378 - 1.1452)^2$
6	$(1.1858 - 1.1571)^2$	$(1.0113 - 1.0592)^2$	$(1.1269 - 1.1452)^2$
7	$(1.1553 - 1.1571)^2$	$(1.0476 - 1.0592)^2$	$(1.1316 - 1.1452)^2$
8	$(1.0952 - 1.1571)^2$	$(1.0935 - 1.0592)^2$	$(1.1466 - 1.1452)^2$
Total	0.0088	0.0094	0.0011

These give a measure of within variation for each machine. How does this jive with the stratified plot?

Within SS: 0.0088 0.0094 0.0011



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To get an overall measure of within variation, compute the **within sum of squares** by adding the sums of squares for the three machines:

$$0.0088 + 0.0094 + 0.0011 = 0.0193.$$

The measure of **between variation**, called the **between sum of squares**, is the difference between the total sum of squares and the within sum of squares:

$$0.0650 - 0.0193 = 0.0457.$$

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Thus, we have

Source	Sum of Squares
Between	0.0457
Within	0.0193
Total	0.0650

From these results, we see that between variation is the principal source of variation for the bakery data, and specifically, that it accounts for

$$100 \times \frac{0.0457}{0.0650} = 70.25\%$$

of the total variation in the data.

SAS code to produce the stratified plot and compute the sums of squares is found here.

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Gage R&R

In a measurement (or gage-ing) process, two sources of variation are often considered:

- **Repeatability**: consistency of the gage (measuring tool) in repeated measurements by the same operator on the same part.
- **Reproducibility**: consistency of measurements between different operators.

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Example:

In a gage R&R study of a laser ranging device, four operators each took fifteen measurements of the same distance. For a gage R&R study, we stratify by operator.

Figure 6 (code here) shows four time series plots, one for each operator, on the same graph. We say the plots are **stratified** by operator.

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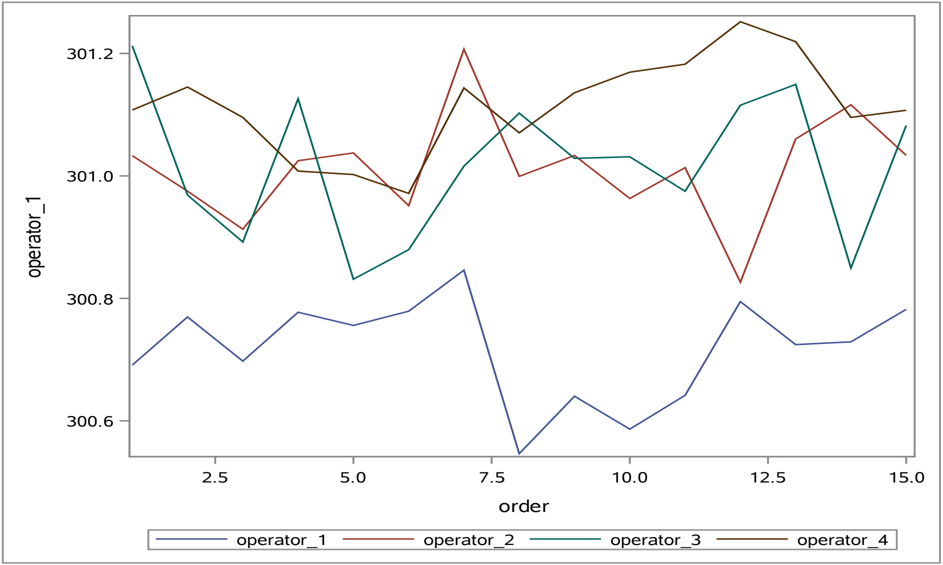

Figure: *Figure 6: Time series plots of laser ranging device readings, stratified by operator.*

None of these plots shows evidence of nonstationarity, which means we can analyze the pattern of variation without regard to order.

Figure 7 (code here) is a stratified plot of the measurements stratified by operator but ignoring order. This is the kind of plot we considered in the bakery example.

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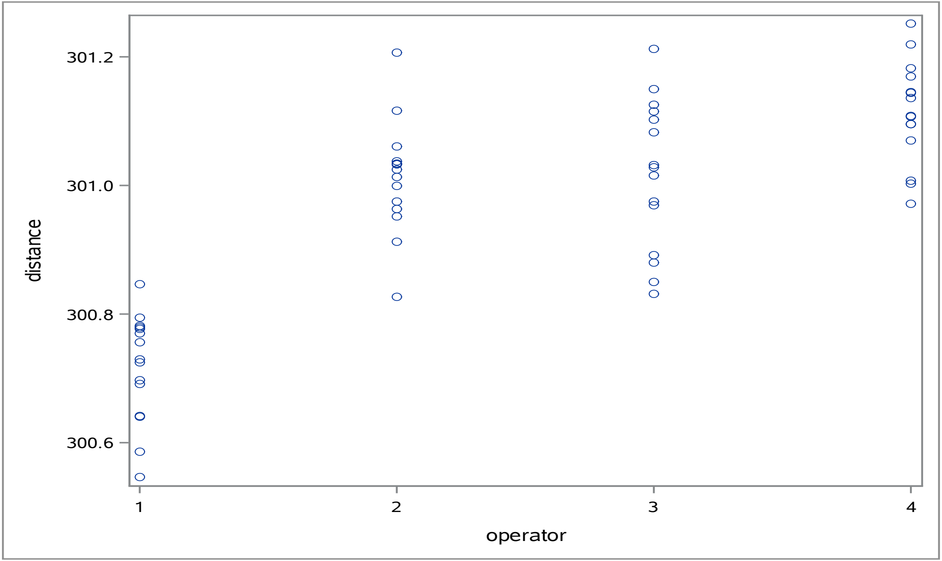

Figure: *Figure 7: Stratified plot of laser ranging device readings, stratified by operator.*

- The within variation measures the **repeatability** of the gage: the consistency of the gage in repeated measurements. Since larger within variation corresponds to the measurement process being less repeatable, it is more accurate to say the within variation measures the non-repeatability.
- The between variation measures the **reproducibility** of the measurement process: the consistency of measurements taken by different operators. As above, it is more accurate to say that the between variation measures the non-reproducibility.

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Consideration of the stratified plot in Figure 7 shows that all operators exhibit roughly the same repeatability, since the spreads of the measurements are roughly the same order of magnitude. In fact the within sums of squares for operators 1 to 4 are 0.0976, 0.1051, 0.1903, and 0.0869, respectively.

Whether the repeatability is adequate for the tasks for which the laser ranging device is used is not answered by this analysis. It can only be answered by the users.

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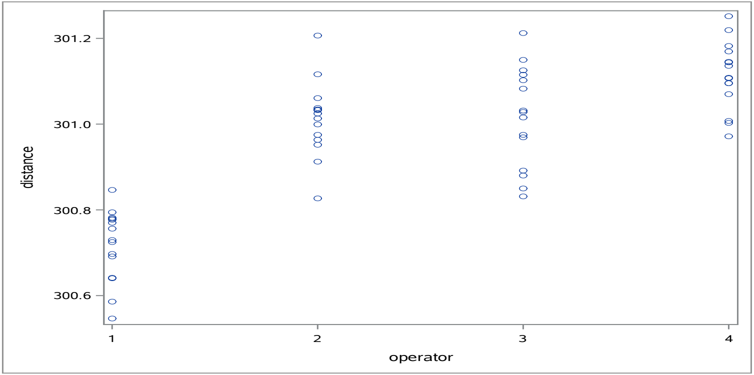
Also, while the measurements of operators 2-4 are centered at roughly the same value, those of operator 1 are centered about a substantially lower value than the other three. This means there is a reproducibility problem with the measuring process.



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To quantify this, we note that the measure of between variation is 1.3258, and that of within variation is 0.4799, giving a total of 1.8058. Thus the percentage of total variation attributable to between variation (or non-reproducibility) is

$$100 \times \frac{1.3258}{1.8058} = 73.42\%.$$

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What's the IDEA?

Variation can sometimes be broken into pieces which identify different sources of the variation and the amount of variation each source contributes. One example is breaking variation into between and within components. Plots of data stratified by these sources can be helpful in analyzing the structure of variation.

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Recap:

- Data and its science, statistics
- Stationary and nonstationary processes; displaying data from each.
- Assessing between and within variation.

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