

Estimating A Population Proportion

One stage of a manufacturing process involves a manually-controlled grinding operation. Management suspects that the grinding machine operators tend to grind parts slightly larger rather than slightly smaller than the target diameter, while still staying within specification limits. To verify their suspicions, they sample 150 within-spec parts and find that 93 have diameters above the target diameter.

If we assume that the parts are independently sampled from a population in which a proportion p have diameters above the target diameter, the number of the 150 parts, y , having diameters larger than the target diameter will have a $b(150, p)$ distribution.

We will use these data to obtain a level 0.99 approximate score confidence interval for p , the true population proportion of parts with diameters greater than the target.

The approximate score interval is computed as follows: Since $L = 0.99$, $z_{(1+L)/2} = z_{0.995} = 2.5758$. Using this in the formula, we obtain

$$\tilde{n} = 150 + 2.5758^2 = 156.6347,$$

and

$$\tilde{p} = \frac{93 + (0.5)(2.5758^2)}{156.6347} = 0.6149,$$

so the interval is

$$\begin{aligned} 0.6149 \pm 2.5758 \sqrt{\frac{0.6149(1 - 0.6149)}{156.6347}} \\ = (0.5148, 0.7151) \end{aligned}$$

99% confidence means that for 99% of all possible samples, intervals computed this way will contain the true population proportion of within-spec parts with diameters above the target diameter.