Comparing Two Population Means: Independent Populations

A company buys cutting blades used in its manufacturing process from two suppliers. In order to decide if there is a difference in blade life, the lifetimes of 10 blades from manufacturer 1 and 13 blades from manufacturer 2 used in the same application are compared. A summary of the data shows the following (units are hours):

Manufacturer	n	$\overline{y}$	s
1	10	118.4	26.9
2	13	134.9	18.4

The investigators found no evidence of nonnormality or outliers in the data. The point estimate of $\mu_1 - \mu_2$ is $\overline{y}_1 - \overline{y}_2 = 118.4 - 134.9 = -16.5$. They decided to compare the mean lifetimes of blades from the two manufacturers by testing the statistical hypotheses

$$\begin{array}{lccc} H_0: & \mu_1 - \mu_2 & = & 0 \\ H_a: & \mu_1 - \mu_2 & \neq & 0 \end{array}$$

A significance level of 0.10 was decided. The manufacturer was unwilling to assume the tow population variances were equal.

Separate Variance Interval

The estimate of the standard error of $\overline{y}_1 - \overline{y}_2$ is

$$\hat{\sigma}(\overline{y}_1 - \overline{y}_2) = \sqrt{\frac{(26.9)^2}{10} + \frac{(18.4)^2}{13}} = 9.92.$$

The distribution of the standardized test statistic is approximated by a t distribution with degrees of freedom ν, computed as the greatest integer less than or equal to

$$\frac{\left(\frac{(26.9)^2}{10} + \frac{(18.4)^2}{13}\right)^2}{\frac{\left(\frac{(26.9)^2}{10}\right)^2}{10-1} + \frac{\left(\frac{(18.4)^2}{13}\right)^2}{13-1}} = 15.17,$$

so $\nu = 15$.

The observed value of the test statistic is

$$t^{(ap)*} = \frac{108.4 - 134.9}{\sqrt{\frac{(26.9)^2}{10} + \frac{(18.4)^2}{13}}} = -2.67.$$

The p-value is computed as

$$p_- = P(t_{15} \leq -2.67) = 0.0087, \; p^+ = P(t_{15} \geq -2.67) = 0.9913,$$

and the p-value for this problem is $p\pm = 2\min(0.0087, 0.9913) = 0.0174$.

The null hypothesis would be rejected at any significance level greater than 0.0174, notably at the 0.10 level. In this case the manufacturer could conclude the mean blade lifetimes of the two manufacturers differ.