Comparing Two Population Means: Independent Populations

A company buys cutting blades used in its manufacturing process from two suppliers. In order to decide if there is a difference in blade life, the lifetimes of 10 blades from manufacturer 1 and 13 blades from manufacturer 2 used in the same application are compared. A summary of the data shows the following (units are hours):

Manufacturer	n	$\overline{y}$	s
1	10	118.4	26.9
2	13	134.9	18.4

The investigators found no evidence of nonnormality or outliers in the data. The point estimate of $\mu_1 - \mu_2$ is $\overline{y}_1 - \overline{y}_2 = 118.4 - 134.9 = -16.5$. They decided to compare the mean lifetimes of blades from the two manufacturers by testing the statistical hypotheses

$$\begin{aligned} H_0: &\quad \mu_1 - \mu_2 &= &\quad 0 \\ H_a: &\quad \mu_1 - \mu_2 &\neq &\quad 0 \end{aligned}$$

A significance level of 0.10 was decided.

Pooled variance test

If the manufacturer is willing to assume the two population variances are equal, a pooled variance test is appropriate. The pooled variance estimate of the common population variance, σ^2 is

$$s_p^2 = \frac{(10-1)(26.9)^2 + (13-1)(18.4)^2}{10+13-2} = 503.6.$$

This gives the estimate of the standard error of $\overline{y}_1 - \overline{y}_2$ as

$$\hat{\sigma}_p(\overline{y}_1 - \overline{y}_2) = \sqrt{503.6\left(\frac{1}{10} + \frac{1}{13}\right)} = 9.44.$$

Under H_0, the standardized test statistic has a t distribution with 10+13-2=21 degrees of freedom. The observed value of the standardized test statistic is $t^{(p)*} = (108.4 - 134.9)/9.44 = -2.81$, from which we can compute

$$p_- = P(t_{21} \leq -2.81) = 0.0052, \; p^+ = P(t_{21} \geq -2.81) = 0.9948,$$

and the p-value for this problem, $p\pm = 2\min(0.0052, 0.9948) = 0.0104$.

The null hypothesis would be rejected at any significance level greater than 0.0104, notably at the 0.10 level. In this case the manufacturer could conclude the mean blade lifetimes of the two manufacturers differ.