Comparing Two Population Means: Independent Populations

A company buys cutting blades used in its manufacturing process from two suppliers. In order to decide if there is a difference in blade life, the lifetimes of 10 blades from manufacturer 1 and 13 blades from manufacturer 2 used in the same application are compared. A summary of the data shows the following (units are hours):

Manufacturer	n	$\overline{y}$	s
1	10	118.4	26.9
2	13	134.9	18.4

The investigators generated histograms and normal quantile plots of the two data sets and found no evidence of nonnormality or outliers. The point estimate of $\mu_1 - \mu_2$ is $\overline{y}_1 - \overline{y}_2 = 118.4 - 134.9 = -16.5$. They decided to obtain a level 0.90 confidence interval to compare the mean lifetimes of blades from the two manufacturers.

Pooled variance interval

The pooled variance estimate is

$$s_p^2 = \frac{(10-1)(26.9)^2 + (13-1)(18.4)^2}{10+13-2} = 503.6.$$

This gives the estimate of the standard error of $\overline{y}_1 - \overline{y}_2$ as

$$\hat{\sigma}_p(\overline{y}_1 - \overline{y}_2) = \sqrt{503.6\left(\frac{1}{10} + \frac{1}{13}\right)} = 9.44.$$

Finally, $t_{21,0.95} = 1.7207$. So a level 0.90 confidence interval for $\mu_1 - \mu_2$ is

$$(-16.5 - (9.44)(1.7207),\ -16.5 + (9.44)(1.7207))$$

$$= (-32.7, -0.3).$$

Separate variance interval

The estimate of the standard error of $\overline{y}_1 - \overline{y}_2$ is

$$\hat{\sigma}(\overline{y}_1 - \overline{y}_2) = \sqrt{\frac{(26.9)^2}{10} + \frac{(18.4)^2}{13}} = 9.92.$$

The degrees of freedom ν is computed as the greatest integer less than or equal to

$$\frac{\left(\frac{(26.9)^2}{10} + \frac{(18.4)^2}{13}\right)^2}{\frac{\left(\frac{(26.9)^2}{10}\right)^2}{10-1} + \frac{\left(\frac{(18.4)^2}{13}\right)^2}{13-1}} = 15.17,$$

so $\nu = 15$. Finally, $t_{15,0.95} = 1.7530$. So a level 0.90 confidence interval for $\mu_1 - \mu_2$ is

$$(-16.5 - (9.92)(1.753),\ -16.5 + (9.92)(1.753))$$

$$= (-33.9, 0.89).$$

There seems to be a problem here. The pooled variance interval, $(-32.7, -0.3)$, does not contain 0, and so suggests that $\mu_1 \neq \mu_2$. On the other hand, the separate variance interval, $(-33.9, 0.89)$, contains 0, and so suggests we cannot conclude that $\mu_1 \neq \mu_2$. What to do?

Since both intervals are similar and have upper limits very close to 0, I would suggest taking more data to resolve the ambiguity.