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(AUTOREGRESSIVE-INTEGRATED MOVING AVERAGE (ARIMA) MODELS
FORECASTING
MOVING AVERAGES
PREDICTION AND FORECASTING
TIME SERIES
TREND
X-11)

ESTELA BEE DAGUM

SECANT METHOD *See* NEWTON–RAPHSON METHODS

SECH-SQUARED DISTRIBUTION *See* LOGISTIC DISTRIBUTIONS

SECONDARY DATA

A term used for data obtained in the course of a survey or experiment designed for some other (primary) purpose. It must be kept in mind that whatever faults there may have been in the planning and execution of the original inquiry can also affect secondary data. In addition, further errors may arise from the way in which the secondary data are extracted from the records of the inquiry.

SECRETARY PROBLEM

The Secretary Problem is one name, among many (see ref. 7), given to a collection of optimal stopping* problems. The two classical Secretary Problems are the best choice problem and the rank problem.

THE BEST CHOICE PROBLEM. It is assumed that n individuals applying for a secretarial position arrive sequentially in random order. Upon arrival each individual is interviewed and ranked (highest rank = 1) relative to all preceding arrivals. At the time of ranking an irrevocable decision must be made to hire or reject the applicant. The goal is to find a strategy to maximize the probability of selecting the best. The optimal strategy (see, e.g., refs. 2 and 7) is to reject the first $s(n)$ individuals and to choose the first, if any, among applicants $s(n) + 1, \dots, n$ who is the best seen so far. It can be shown that $s(n)/n \to e^{-1} \doteq 0.368$ and that this is also the optimal asymptotic probability of selecting the best.

THE RANK PROBLEM. (See refs. 1 and 9.) The mechanics are the same as in the best choice problem, but here the goal is to find a strategy to minimize the expected rank of the applicant chosen. There is a nondecreasing sequence of integers $\{s(n; k), 1 \leqslant k \leqslant n\}$ such that the optimal strategy is of the form: if no selection has been made from the first $s(n; k) - 1$ applicants, select the first arrival among applicants $s(n; k), \dots, s(n; k + 1)$ who is one of the k best seen so far. Chow et al. [1] show that the optimal asymptotic expected rank is approximately 3.87.

The origins of the Secretary Problem are unclear. According to ref. 7, in 1955

Frederick Mosteller was told the best choice problem by Andrew Gleason who had heard it from someone else. Fox and Marnie posed what is basically the best choice problem under the name "Googol" in the Mathematical Games column of the February, 1960 *Scientific American*. A solution was outlined by Moser and Pounder in the same column the following month.

Since that time the Secretary Problem has generated a substantial literature. Among the reasons for its popularity may be: it is simple to state; its solutions are intuitively appealing; it touches on the disciplines of probability, statistics, and operations research*; and it is a model for a common real-life problem that invites improvement and generalization. This last may explain why the development of the Secretary Problem up to the present has consisted primarily of generalizations designed to make the two basic models more realistic. A description of some of these follows; others are briefly described in the annotated bibliography.

FULL AND PARTIAL INFORMATION PROBLEMS

The above strategies are based only on rankings of the applicants. In some cases it may be reasonable to assume more information about the applicants—for example, one may observe scores on placement tests. In such a case the observations could be considered independent identically distributed (i.i.d.) random variables from a known distribution (full information) or from a family of distributions (partial information). Hereafter "no information" will describe problems in which only ranks are observed.

The object may be to maximize the probability of best choice, the expected quantile of the chosen observation (the analog of the rank problem), or the expectation of some other payoff function of the chosen observation.

Gilbert and Mosteller [7] solved the full information best choice and quantile problems. The asymptotic optimal probability of best choice in the former is approximately 0.58, while the asymptotic minimal expected rank in the latter equals 2.

The earliest work on the partial information problem [3, 19, 25] assumed a normal distribution with unknown mean and a payoff equal to the value of the chosen observation minus a sampling cost. Later Stewart [23] and Samuels [20] considered the best choice and quantile problems when observations are from a uniform* $U(\alpha, \beta)$ distribution with α and β unknown. The latter paper showed that for the best choice problem the minimax* rule is simply the no information rule for the classical best choice problem, while for the quantile problem the minimax rule gives an asymptotic expected rank of approximately 3.478.

For the best choice problem, Petruccelli [14] obtained sufficient conditions on location–scale parameter families of distributions for the existence of minimax rules, which asymptotically attain the asymptotic full information probability of best choice. One family that satisfies these conditions is the family of all normal distributions.

Petruccelli [10] also showed that for the family of $U[\theta - \frac{1}{2}, \theta + \frac{1}{2}]$ distributions with θ unknown, the minimax rule in the best choice problem has asymptotic probability of best choice of approximately 0.435—intermediate between the full and no information values.

Enns [4] studied problems with a different kind of partial information: the knowledge of whether or not an observation exceeds a certain value.

PROBLEMS INVOLVING BACKWARD SOLICITATION AND / OR UNCERTAINTY OF CHOICE

Yang [26] modified the classical best choice problem to allow solicitation of a past observation. He assumed that just after applicant k was interviewed, applicant $k - r$ would accept an offer with probability $q(r)$.

This model assumed $q(0) = 1$; that is, an applicant is sure to accept an immediate offer. Smith and Deely [22] assumed (in Yang's terminology) that $q(r) = 1$, $0 \leqslant r \leqslant m$, and $q(r) = 0$, $r > m$; that is, recall memory lasts m interviews.

Smith [21] considered the case of uncertain choice without recall that corresponds to $q(0) < 1$ and $q(r) = 0$, $r \geqslant 1$, in Yang's terminology. Petruccelli [11] combined the models of Yang and Smith by allowing $q(0) < 1$ in Yang's formulation. In addition Petruccelli [12] extended backward solicitation and uncertainty of choice to the full information best choice problem. In this last paper, recall probabilities were functions of the quantile of the observation as well as the time since observation.

While the models described here allow for a more realistic formulation than does the classical best choice problem, there is a price to be paid: it is impossible to obtain closed-form optimal rules without further assumptions on the recall probabilities.

A RANDOM NUMBER OF OBSERVATIONS

Secretary problems with a random number of observations N are in general complicated and admit no closed form solution. For the classical best choice problem in which the number of observations is a bounded random variable, Petruccelli [13] showed that after allowing for an initial learning period, the optimal rule can take literally any form.

Among generalizations for arbitrary N, Presman and Sonin [16] obtained results for the best choice problem, while Irle [8] considered an arbitrary payoff.

For bounded N Rasmussen [17] studied general payoffs based on ranks, Rasmussen and Robbins [18] considered the best choice problem, Gianini-Pettit [6] dealt with expected rank payoff, and Petruccelli [15] added backward solicitation and uncertainty of choice to the best choice problem. Stewart [24] used a Bayesian approach to the random N problem in which arrivals were assumed to occur at i.i.d. exponential intervals.

OTHER GENERALIZATIONS

There are many other generalizations of the Secretary Problem: some are indicated in the Bibliography; a number are found in the Gilbert and Mosteller paper [7], which even after nearly two decades remains a good introduction to the topic. The best introduction to the development of the problem over the past 20 years is a review article by Freeman [5].

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(OPTIMAL STOPPING RULES)

Joseph D. Petruccelli

SECTOR CHART; SECTOR DIAGRAM

See PIE CHART

SECULAR TREND *See* TREND

SEEMINGLY UNRELATED REGRESSION

Consider the set of M regression models

$$y_j = X_j\beta_j + u_j, \qquad j = 1, \dots, M, \quad (1)$$

where y_j is a $T \times 1$ vector of observations on the jth dependent variable, X_j is a $T \times K_j$ matrix of observations on K_j nonstochastic regressors assumed to be of full column rank, β_j is a $K_j \times 1$ vector of regression coefficients, and u_j is a $T \times 1$ vector of random disturbances with $E(u_j) = 0$ and $E(u_j u_i') = \sigma_{ji} I_T$ for $i, j = 1, \dots, M$. If the observations correspond to different points in time, the specification implies that the disturbances in different equations are correlated at each point in time but are uncorrelated over time. Variances and covariances remain constant over time. We further assume that the disturbances are distributed independently over time and that the matrix of sample moments of all distinct regressors in (1) converges to a