Saving the calculating time of the TCNN with nonchaotic simulated annealing strategy

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Abstract—**The Transient Chaotic Neural Network (TCNN) and the Noisy Chaotic Neural Network(NCNN) have been proved their searching abilities for solving combinatorial optimization problems(COPs). The chaotic dynamics of the TCNN and the NCNN are believed to be important for their searching abilities. However, in this paper, we propose a strategy which cuts off the rich dynamics such as periodic and chaotic attractors in the TCNN and just utilizes the nonchaotic converge dynamics of the TCNN to save the time needed for computation. The strategy is named as nonchaotic simulated annealing (NCSA). Experiments on the traveling salesman problems exibit the effectiveness of NCSA. The NCSA saves over half of the time needed for the computation while maintaining the searching ability of the TCNN.**

Keywords—**TCNN, CSA, nonchaotic, NCSA , combinatorial optimization problems, TSP**

I. INTRODUCTION

Since Hopfield and Tank proposed their neural networks and mapped the traveling salesman problem(TSP) into their neural networks based on the energy function method, neural network methods for combinatorial optimization problems have been intensively studied(e.g. see[1]-[20]).

In recent years, the research on chaotic neural networks draws a lot of attention since chaotic dynamics was found in real neurons and real neural networks[1]. Chaotic neural networks have rich dynamic behaviors such as periodic dynamics and chaotic dynamics. And they have shown a better solving ability than the conventional Hopfield Neural Networks for the TSP[1][4][5].

Chen and Aihara proposed a transiently chaotic neural network(TCNN,1995) with chaotic simulated annealing strategy(CSA) based on the chaotic neural network[7].The stronger searching ability of the TCNN shows that it's an useful tool for combinatorial optimization problems. A series of works had been done to investigate the TCNN. Chen and Aihara proved that the TCNN will be chaotic when absolute values of the self- feedback connection weights are sufficiently large and that TCNNs do have strange attractors[13],[15].They also theoretically explained that the TCNN has a global attracting set which contains all the global minima, so the global search can be carried out[14]. Zhao and Lin etc. drove the sufficient condition for the stability of TCNN[18]. Inspired by the TCNN, new methods of chaotic simulated annealing with transient chaotic dynamics were proposed[8][9].Kwok and Smith proposed a unified energy framework for three chaotic models and investigate them based on the unified framework[10]. Also a Noisy Chaotic

Neural Network (NCNN) was proposed by Lipo Wang et al. by adding decaying stochastic noises to the TCNN[19]. The NCNN performances as well as the TCNN in relatively small scale problems and offers better results than the TCNN in larger scale problems. The above two models were applied to solving a series of NP-hard problems such as broadcast scheduling problems in packet radio networks and frequency assignment problems in satellite communications. Better results are gained compared with former neural network models[20],[21],[22].

While being applied to COPs, the TCNN and the NCNN gradually approach, through transient chaos, to a stable equilibrium point by controlling a bifurcate parameter. It's well believed that chaotic dynamics and chaotic attractors play an important role in their applications to optimization problems[7],[13]-[15],[22]. However, in this paper, we suggest a new strategy that we only use the nonchaotic asymptotically converge dynamic of the TCNN. In the dynamic progress, there will be no chaotic attractors appeared. Experimental results on TSPs show that our method saves over half of the time needed for calculations compared with the traditional TCNN, which will be a big advantage when the TCNN is applied to problems with large scales. Compared with the CSA, the new strategy is named nonchaotic simulated annealing(NCSA). The structure of the paper is organized as follows. In Section II, the mechanism of the TCNN is discussed and the TCNN is concluded as a special continuous Hopfield Neural Networks(CHNN). In Section III, numerical analysis and experimental results on 10-city, 21-city and 52-city TSPs with the CSA and the NCSA are compared and analyzed.

II. TCNN

We get the TCNN by adding a negative self-feedback term $z_i^{'}(v_i - I_0)$ to the continuous Hopfield Neural Network and decreasing the self-feedback neural connection weights $z_i^{'}$ exponentially during the dynamic progress:

$$\frac{du_i}{dt} = -\frac{u_i}{\tau} + (\sum_{j=1}^{r} w_{ij}v_i + I_i) - z_i^{'}(v_i - I_0) \quad (1)$$

$$\frac{dz^{'}}{dt} = -\beta^{'}z^{'} \quad (2)$$

$$v_i = \frac{1}{1 + e^{-u_i/\varepsilon}} \quad (3)$$

And the difference equations based on the Euler method constitute the TCNN[7]:

$$u_i(t+1) = ku_i(t) + \alpha(\sum_{j=1}^{n} w_{ij}v_j(t) + I_i) - z(t)(v_i(t) - I_0) \tag{4}$$

$$i = 1, 2, ..., n$$

$$v_i(t+1) = \frac{1}{1+e^{-u_i(t)/\varepsilon}} \tag{5}$$

$$z_i(t+1) = (1-\beta)z_i(t) \tag{6}$$

where the variables for equations(1)-(3) are:

u_i internal state of neuron i;

τ time constant of CHNN;

w_{ij} connection weight from neuron j to neuron i;

I_i input bias of neuron i;

I_0 a positive parameter

ε steepness parameter of the neural output function;

z_i' the self-feedback neural connection weight;

β' the damping factor of z_i';

The relations between the continuous model and discrete model are:

$$k = 1 - \frac{\Delta t}{\tau}; \ \alpha = \Delta t; \ z(t) = \Delta t z'(t); \ \beta = \Delta t \beta'$$

where Δt is the time step by the Euler method. According to equations(1)-(6), we can rewrite the dynamic equation of the TCNN as (7) and summarize TCNN as a special CHNN:

$$\frac{du_i}{dt} = -\frac{u_i}{\tau} + (\sum_{j=1}^{r} w_{ij}'v_i + I_i') \tag{7}$$

where $w_{ij}' = \begin{cases} w_{ij} & i \neq j \\ w_{ij} - z' & i = j \end{cases}, I_i' = I_i + z_i'I_0$.

It's well known that CHNN utilizes a gradient descent dynamic for the optimization problems as shown in(8):

$$\frac{du_i}{dt} = -\frac{u_i}{\tau} - \frac{\partial E}{\partial v_i} \tag{8}$$

Thus, we can see that the TCNN also utilizes a gradient descent dynamic, though when the time step and the self-feedback neural connection weights are sufficiently large, the system shows rich dynamics such chaotic attractors.
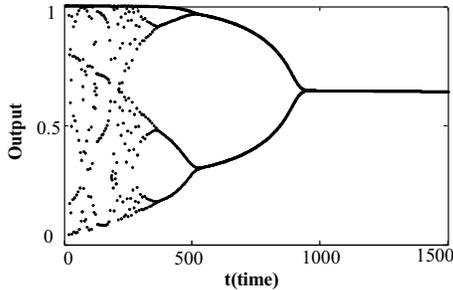

Figure 1. Time evolution of the output of a single neuron

III. NUMERICAL ANALYSIS

A. Dynamics of single neuron

Positive Lyapunov exponent has been observed from the single-neuron model of TCNN, which means the system is chaotic[7]. From Fig. 1, we can see that the output of a neuron gradually transits from chaotic behavior to a fixed point through reversed period-doubting bifurcations, where the last bifurcate point sits at about iteration 900. In our NCSA, we intend to start the dynamics of the TCNN from the last bifurcate point, and the chaotic dynamics are removed, when the TCNN is applied to COPs.

B. Dynamics of TCNN for the TSP

Next, we use the TSP to explain our method. Symmetric TSPs are adopted. Using the same method of Hopfield and Tank[1], we form a n-dimension matrix using the outputs of TCNN to represent the solution for the TSP and propose the energy function for a n-city TSP as follows:

$$E = \frac{W_1}{2}(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k \neq j}^{n} v_{ij}v_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l \neq i}^{n} v_{ij}v_{lj} - 2\sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij})$$
$$+ \frac{W_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} v_{ij}d_{ik}(v_{kj+1} + v_{kj-1}) \tag{9}$$

The first term with a coefficient $W_1/2$ guarantees the matrix only has one "1" in each row and each column and the rest elements are all "0" when the term is at its global minimum or its local minimum. We get dynamic equations of TCNN for the TSP as(11) using(10)[1]:

$$E(v_i) = -\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} w_{ij}v_iv_j - \sum_{i=1}^{n} v_iI_i \tag{10}$$

$$u_{ij}(t+1) = ku_{ij}(t) - z(t)(v_{ij}(t) - I_0) - $$
$$\alpha\left\{ W_1\sum_{k \neq j}^{n} v_{ik}(t) + W_1\sum_{l \neq i}^{n} v_{lj}(t) - W_1 \right.$$
$$\left. + W_2\sum_{k=1}^{n} d_{ik}\left(v_{k,j+1}(t) + v_{k,j-1}(t)\right) \right\} \tag{11}$$

Combining (5)(6)(9), we get the equations for solving the TSP. The NCNN model can be get if we add a decaying stochastic noise to the TCNN, with the initial amplitude and decaying speed being chosen independently. The first four cities of Hopfield-Tank 10-city TSP are used to examine the chaotic dynamic of TCNN. The parameters are selected the same as Chen and Aihara's:

$k = 0.9$; $\varepsilon = 1/250$; $I_0 = 0.65$; $z(0) = 0.08$; $W_1 = 1$, $W_2 = 1$; $\alpha = 0.015$; $\beta = 0.001$.

where $z(0)$ is the initial value of self-feedback neural connection weight. The states of neurons are cyclically updated.

Fig. 2 shows the time evolution of the energy function, neural outputs and the damping of z with $z(0) = 0.10$. The dynamics of the network are understood to be chaotic mostly during the first 800 iterations[7]. And after iteration 800, the dynamics go to the asymptotical stable phase. And around iteration 1100, where $z < 0.02$, the system rapidly transits

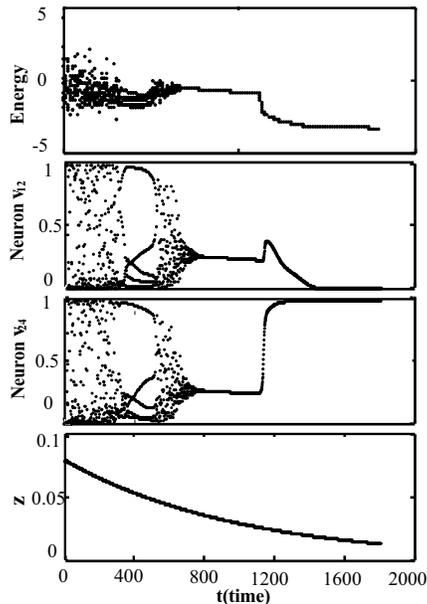from the non-integer solution through falls toward a nearly integer solution.
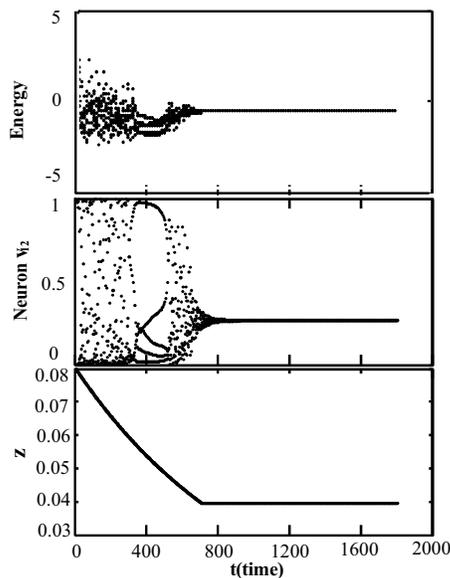

Figure 2. The dynamic of the TCNN with $z(0) = 0.10$

Figure 3. The neural networks dynamics (stop the damping of z when $z < 0.0395$)

Fig. 3 shows that the TCNN asymptotically converges to a fixed point immediately after we stop decreasing z when $z \leq 0.0395$, and the time needed to be stable is about one or two iterations. In this four-city case, TCNN asymptotically converges to a non-integer fixed point during about iteration 800 to iteration 1100.
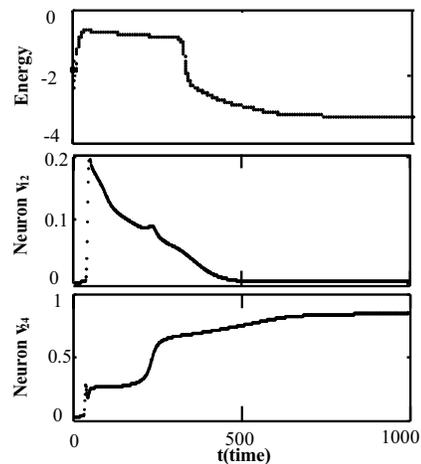

Figure 4. The time evolution of TCNN with relatively small initial value of self-feedback neural connection weight. z(0)=0.036

In previous works, a sufficient large $z(0)$ was used so that the neural networks shows chaotic behaviors, and the strategy was called chaotic simulated annealing [7],[19]. However, in this paper, we propose that we start from the last bifurcate point and just utilize the asymptotical stable search phase by using a proper $z(0)$ as shown in Fig. 4. In contrast with CSA, we call it nonchaotic simulated annealing(NCSA).

C. Solving the TSP with NCSA

We first choose the original Hopfield-Tank 10-city TSP to illustrate the effectiveness of our proposal. The accelerating method we used is the same with Chen and Aihara's in[7]. If the discrete output is a valid solution and keeps unchanged within 50iterations, then the calculation is terminated and the results are calculated. The first set of parameters for TCNN are set the same with Chen and Aihara's with $\beta = 0.003$, while we suggest using a smaller initial value of self-feedback weight: $z(0) = 0.04$ for the second set. Repeating the simulations with 1000 different initial states u_{ij} generated randomly in the region [-1,1], we get the results shown in Table I. The time evolutions of the neuron outputs of two methods are shown in Fig. 5.
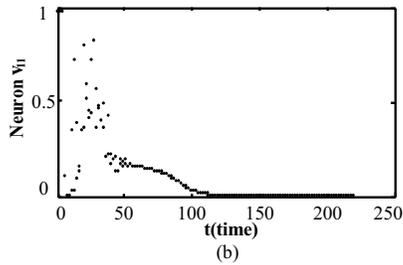

(a)
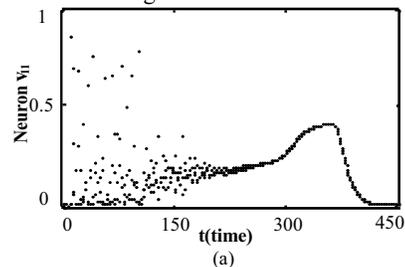
Figure 5. A comparison of neural output V(1,1) of two kinds of method:
(a)the Chen and Aihara's method;(b)our method.

TCNN	CSA $z(0) = 0.08$	NCSA $z(0) = 0.04$
Number of global minima	1000(100%)	1000(100%)
Number of others	0	0
Average iterations	448	218.1

Fig. 5(b) shows that using a smaller $z(0)$ decreases a lot of chaotic dynamics. The dynamics of neuron V(1,1) is unstable during the first 40 iterations, then it turns to a asymptotical stable phase and stays at 0 finally. Noticing that in order to guarantee the optimal percentage, we start network dynamic a bit earlier than the bifurcate point. From Table I we can find that the both CSA and NCSA find the optimal route with 100 percent percentage. The iterations needed of our method is 218.1 while the calculation of CSA need 448. About 1/2 of the computing time is saved, but the ability of the TCNN is not affected.

Next, we use the gr21 from Lipo Wang's work for comparison and illustration. The optimal tour length is 2707 in TSBLIB. And $z(0) = 0.04$ is adopted. The other parameters are the same with Lipo Wang's[18]. Because the distances are relatively large in gr21, we need to normalize the distances to the range [0,1] before computation and covert them back while calculating the results so that the second term in the Energy function won't weight too much. The 100 results with different initial internal states u_{ij} generated randomly in the region [-1,1] is shown in Table II. A comparison of the energy functions of CSA and NCSA is shown in Fig. 6, and Fig.7 is an amplification of Fig. 6(a).

Fig. 7 and Fig.6(a) show that during the first 80 iterations the system of NCSA is unstable with periodic behavior. Then the rest are. monotonously gradient descent dynamics. The chaotic dynamics is nearly eliminated, but the computing progress still founds the optimal solution every time according to Table II. On the contrary, Fig. 6(b) shows the rich dynamic

of TCNN with relatively large self-feedback neural connection weights in the first stage. From Table II, we can see that the time needed of our method is only about 39% of Lipo Wang's method, which is a great improvement.
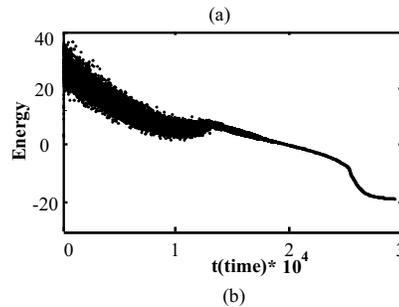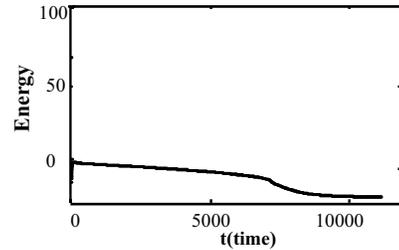

(a)

(b)

Figure 6. A comparison of energy of two kinds of methods:
(a)NCSA;(b)CSA.
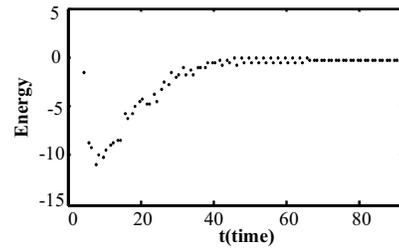

Figure 7. Local amplification of Fig. 2(a)

	Lipo Wang's method		NCSA
	TCNN	NCNN	
$z(0)$	0.10	0.10	0.04
Number of global minima	100	100	100
Average iterations	29105	27578	11160

Further more, Berlin52 from the TSPLIB is adopted. The best tour length for Berlin52 in TSPLIB is 7542. Also the distances are normalized before calculation. We repeat the simulations with 100 different initial internal states generated randomly in the region [-1,1]. The first set of parameters of TCNN are as follows：

$$k = 0.9 , \quad \varepsilon = 1/250 , \quad I_0 = 0.65 ; \quad W_1 = 1 , \quad W_2 = 1 ;$$
$$\alpha = 0.015 ; \quad \beta_1 = 0.00002 ; \quad z(0) = 0.10 .$$

And the second set use a different $z(0) = 0.04$. Table III summarizes the results for the Berlin52 TSP. It shows that the CSA and NCSA get equally good results while NCSA only uses 1/3 time of the other. It does indicate that NCSA saves a lot time needed for the computation than CSA.

TABLE III.　　COMPARISON OF TCNN WITH CSA AND NCSA ON BERLIN52 TSP FOR 100 RUNS WITH RANDOM INITIAL STATES OF THE NETWORKS

TCNN	CSA	NCSA
	$z(0) = 0.10$	$z(0) = 0.04$
Best tour length	7632	7598
Number of valid	100	100
Average length	8047	8014.9
Average iterations	67219	21239

IV. CONCLUSION

In this paper, we review the TCNN as a special CHNN and use three cases of the TSP to explain that the TCNN can maintain its searching ability while the chaotic dynamics is removed. Cutting the chaotic dynamics saved a lot of time needed for computation. This is a new result since chaotic attractors are emphasized for the searching ability of the TCNN. It reminds us to look into the searching mechanism of the TCNN. Because NCNNs are generally based on TCNNs, it's well believed that the chaotic dynamics of NCNNs can also be removed. And that will be our future works.

REFERENCES

[1] C. A. Skarda and W. J. Freeman, "How brains make chaos in order to make sense of the world," Brain Behav. Sci., vol. 10, pp. 161–195, 1987.

[2] J. J. Hopfield & D. Tank, "Neural computation of decisions in optimization problems," Biolog. Cybern.,vol. 52, pp.141-152, 1985.

[3] Shigeo Abe,"Global Convergence and Suppression of Spurious States of the Hopfield Neural Networks". IEEE Trans. Circuits Syst.I: vol. 40. no. 4, 1993.

[4] K. C. Tan, Huajin Tang, and S. S. Ge,"On Parameter Settings of Hopfield Networks Applied to Traveling Salesman Problems", IEEE Trans. Circuits Syst.I: vol. 52, no. 5, 2005.

[5] N. Funabiki and S. Nishikawa, "A gradual neural-network approach for frequency assignment in satellite communication systems," IEEE Trans. Neural Networks, vol. 8, no. 6, pp. 1359–1370, Nov. 1997.

[6] Nozawa.H, "A neural network model as a globally coupled map and applications based on chaos," Chaos, 2(3), 377-386, 1992.

[7] L.Chen and K.Aihara, "Chaotic Simulated Annealing by a Neural Network Model with Transient Chaos," Neural Networks, vol. 8, no. 6, pp. 915-930, 1995.

[8] L. Wang and K. Smith, "On chaotic simulated annealing," IEEE Trans.Neural Networks, vol. 9, pp. 716–718, July 1998.

[9] Y. He, "Chaotic Simulated Annealing With Decaying Chaotic Noise", IEEE Trans. Neural Networks, vol. 13, no. 6,pp.1526-1531, July 2006.

[10] T. Kwok and K. A. Smith, "A unified framework for chaotic neural network approaches to combinatorial optimization," IEEE Trans. Neural Networks, vol. 10, no. 4, pp. 978–981, 1999.

[11] T. K.wok and K.A. Smith, "Experimental analysis of chaotic neural network models for combinatorial optimization under a unifying framework", Neural Networks, vol.13, pp.731-744, 2000.

[12] I. Tokuda, K. Aihara, and T. Nagashima, "Adaptive annealing for chaotic optimization", Phys. Rev. E, vol. 58,no. 4, pp.5157-5160, 1998.

[13] L. Chen and K. Aihara, "Chaos and asymptotical stability in discrete-time neural networs," Physica D, vol. 104, no. 3/4, pp. 286–325,1997.

[14] L. Chen and K. Aihara, "Global searching ability of chaotic neural networks" ,IEEE Trans. Circuits Syst. I, vol. 48, pp. 974–993, Aug, 1999.

[15] L. Chen and K. Aihara, "Strange Attractors in Chaotic Neural Networks", IEEE Trans. Circuits Syst. I, vol. 47, no.10, pp.1455–1468, 2000.

[16] M.Hasegawa, T.Keguchi, K.Aihara,"Solving large scale traveling salesman problems by chaotic neurodynamics", Neural Networks, vol.15, pp. 271-283, 1997.

[17] I. Tokuda, T. Nagashiman and K. Aihara, "Global Bifurcation Structure of Chaotic Neural Networks and its Application to Traveling Salesman Problems", Neural Networks, vol. 10, no.9, pp. 1673-1690, 1997.

[18] W. Zhao, W. Lin, R. Liu and J. Ruan, "Asymptotical stability in discrete-time neural networks," IEEE Trans. Circuits Syst.I, vol. 49, no. 10, pp. 1516-1520, Oct. 2002.

[19] L. P. Wang, S. Li, F. Y. Tian, and X. J. Fu, "A noisy chaotic neural network for solving combinatorial optimization problems: stochastic chaotic simulated annealing," IEEE Trans. Syst. Man Cybern. B, Cybern. vol. 34, no. 5, pp. 2119–2125, 2004.

[20] L. P. Wang and H. Shi, "A Gradual Noisy Chaotic Neural Network for Solving the Broadcast Scheduling Problem in Packet Radio Networks," IEEE Trans. Neural Networks, vol. 17, no. 4, pp.989-1000, Jul. 2006.

[21] L. P. Wang, W. Liu and H. Shi, "Noisy Chaotic Neural Networks with Variable Thresholds for the Frequency Assignment Problem in Satellite Communications," IEEE Trans. Syst., Man, Cybern. C, Appl. Rev. vol. 38, no. 2, pp.209-217, Mar. 2008.

[22] Z. He, Y.Zhang, C.Wei, and J.Wang, "A Multistage Self-Organizing Algorithm Combined Transiently Chaotic Neural Network for Cellular Channel Assignment", IEEE Trans. Veh. Technol., vol. 51, no. 6, Nov. 2002.