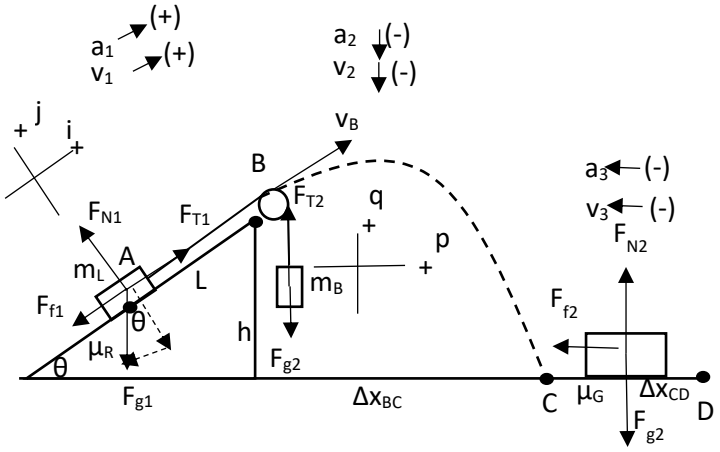
Justin Che 11/03/2021 Section S

The Problem: Leaping Larry decided to make a laborious launcher for his luxury luge using a pulley and ramp system (see diagram). His method was to attach one end of a massless stretchless rope to a barrel of rocks and to hold the other end of the rope. He placed the rope over a massless frictionless pulley, and then walked down the ramp far as down possible to point A (where L = h). When he sat in the luge he accelerated up the ramp to point B and then launched off the top at the same angle as the ramp (all while releasing the rope and avoiding the pulley). He flew through the air as a projectile to point C, smoothly transitioning all of his (net) speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any height differences between luge height, barrel height, and size of the pulley, and the diagram is not drawn to scale.



Assume:

$F_{T1} = F_{T2}$

$a_1 = -a_2$

Givens:

$m_L = 28kg$

$m_B = 44kg$

$\theta = 37^{\circ}$

$\mu_R = 0.17$

$L = 8.6m$

$h = 8.6m$

$\Delta x_{BD} = 55m$

$\mu_G = ??$

Step 1: Finding V_B using pulley. First need to find acceleration

$$\sum F_i: F_{T1} - F_{f1} - F_{g1}sin\theta = m_L a_{x1}$$

$$\underline{F_{T1} = \mu_R F_{N1} + F_{g1}sin\theta + m_L a_{x1}}$$

0 acceleration in y-DIR

$$\sum F_j: F_{N1} - F_{g1}cos\theta = m_L a_{y1}$$

$$F_{N1} = m_L g sin\theta$$

$$F_{N1} = 28 * 9.8 * cos37$$

$$\underline{F_{N1} = 219.146N \nwarrow}$$

$$\sum F_q: F_{T2} - F_{g2} = m_B a_{y2}$$

$$\underline{F_{T2} = m_B a_{y2} + m_B g}$$

Set $F_{T1} = F_{T2}$

$$\mu_R F_{N1} + F_{g1}sin\theta + m_1 a_1 = -m_B a_1 + m_B g$$

$$\mu_R F_{N1} + m_L g sin\theta + m_1 a_1 + m_B a_1 = m_B g$$

$$m_1 a_1 + m_B a_1 = -\mu_R F_{N1} - m_L g sin\theta + m_B g$$

$$a_1(m_L + m_B) = -\mu_R F_{N1} - m_L g sin\theta + m_B g$$

$$a_1 = \frac{-\mu_R F_{N1} - m_L g sin\theta + m_B g}{m_L + m_B}$$

$$a_1 = \frac{-0.17 * 219.146 - 28 * 9.8 * sin37 + 44 * 9.8}{28 + 44}$$

$$a_1 = \frac{-37.2547 - 165.138 + 431.2}{72}$$

$$a_1 = \frac{228.807}{72}$$

$$\underline{a_1 = 3.17788m/s^2}$$

Step 2: finding V_B using kinematics EQ4

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_B^2 = v_A^2 + 2a_1 L$$

Object begins at rest, so $V_A = 0m/s$

$$v_B^2 = 2 * 3.178 * 8.6$$

$$v_B^2 = 54.6595$$

$$\underline{v_B = 7.3932m/s}$$

Step 3: using x-DIR y-DIR to find Δx_{BC}

$$\underline{v_{By} = v_B sin\theta}$$

$$\underline{v_{Bx} = v_B cos\theta}$$

x-DIR: $$x_f = \frac{1}{2}at^2 + v_i t + x_i$$

0 acceleration in x-DIR

$$x_C = v_{Bx}t + x_B$$

$$\underline{x_C = v_B cos\theta t + x_B}$$

y-DIR: $$x_f = \frac{1}{2}at^2 + v_i t + x_i$$

$$y_C = \frac{1}{2}at^2 + v_{By}t + y_B$$

$$y_C = \frac{1}{2}at^2 + v_B sin\theta t + y_B$$

$$0 = \frac{1}{2} * -9.8 * t^2 + 7.393\sin(37)t + 8.6$$

$$0 = -4.9 * t^2 + 4.45t + 8.6, \text{ Solver}$$

$$\underline{t = \cancel{-0.946s} \text{ or } t = 1.85455}$$

x-DIR:

$$x_C = v_B cos\theta t + x_B$$

$$\Delta x_{BC} = v_B cos\theta t$$

$$\Delta x_{BC} = 7.393 * cos37 * 1.8545$$

$$\underline{\Delta x_{BC} = 10.945m}$$

Step 4: Finding Δx_{CD}

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$

$$\Delta x_{CD} = 55 - 10.954$$

$$\underline{\Delta x_{CD} = 44.06\text{m}}$$

Step 5: Finding total net transfer of velocity

$$\underline{v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2}}$$

$$v_{Cx} = v_{Bx}$$

$$v_{Cx} = v_B cos\theta$$

$$v_{Cx} = 7.39cos37$$

$$\underline{v_{Cx} = 5.90447m/s}$$

Continued

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_{Cy}^2 = v_{By}^2 + 2a\Delta y$$

$$v_{Cy}^2 = (v_B sin\theta)^2 + 2 * -9.8 * -8.6$$

$$v_{Cy}^2 = (7.39sin37)^2 + 2 * -9.8 * -8.6$$

$$v_{Cy}^2 = 188.339$$

$$\underline{v_{Cy} = 13.7243m/s}$$

$$v_C = \sqrt{5.90447^2 + 13.7243^2}$$

$$\underline{v_C = 14.9405m/s}$$

Step 6: Finding a_C using Kinematics EQ4

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_D^2 = v_C^2 + 2a_C\Delta x_{CD}$$

$$0 = v_C^2 + 2a_C\Delta x_{CD}$$

$$a_C = \frac{-v_C^2}{2\Delta x_{CD}}$$

$$a_C = \frac{-14.94^2}{2 * 44.06}$$

$$a_C = \frac{-223.17}{88.11}$$

$$\underline{a_c = -2.53393m/s^2}$$

Final Step: Finding μ_G

$$\sum F_x: -F_{f2} = ma_C$$

$$-F_{f2} = 28 * -2.53393$$

$$-F_{f2} = 28 * -2.53393$$

$$\underline{F_{f2} = 70.9502N \leftarrow}$$

$$\underline{\cancel{F_{f2} = \mu_G F_{N2}}}$$

$$F_{N2} = mg$$

$$F_{N2} = 28 * 9.8$$

$$\underline{F_{N2} = 274.4N \uparrow}$$

$$70.9502 = \mu_G 274.4$$

$$\mu_G = \frac{70.9502}{274.4}$$

$$\boxed{\mu_G = 0.2586}$$