2021 HiMCM Problem A: Storing The Sun

Team 11618

1 Summary

Climate change poses an existential threat to humanity due to the potential for massive disruptions in agriculture, geography, housing, immigration, and the economy. The current dependency on fossil fuels and a centralized electrical grid precipitates long term negative consequences. Thus, sustainable solutions are needed. Harnessing solar power to meet homeowners' energy needs is a sustainable method of powering a home, but is often difficult due to the limitations of solar storage batteries. This issue becomes especially important when a homeowner lives completely off the grid, and must derive all of their home's power from a battery storage system. In this paper we develop two comprehensive models for selecting the best solar battery system for a 1600 square foot home being outfitted with solar panels, with no access to a power grid. We then extend this model to be applicable to homes of any size.

In response to Problem A, two unique solutions are modeled. The first, which we will refer to as Model 1, the "Single Battery Brand Model," utilizes an algorithmic ranking criteria to output the optimal number of batteries of the same brand, given the homeowner's budget and the number of people living at the home. This is a simpler model, because it does not account for combinations of batteries, but it is a good method for ranking batteries based on variable energy needs. We were able to both specify it to the home in the problem, as well as generalize it, but it is not as dynamic as Model 2.

Model 2, called the "Functional Combinations Model" is significantly more sophisticated than Model 1. It utilizes a cyclic trigonometric model (with a period of 24 hours) for the house's power needs, including an estimate of when the power needed is higher than average, and when the power needed is lower than average. In addition, it also uses multiple power regression models to determine non-linear continuous battery output rate decay of certain batteries. Model 2 is also designed to account for the usable capacity criteria in relation to energy demand, so that the solution the model yields is feasible. The model is able to generate an optimal combination of batteries given a certain amount of "blackout" time - the length of time without any solar panel energy input - by cycling through every combination of batteries. We then adjust our equations to generalize model 2 for different house sizes, in order to optimize the battery storage for any sized home.

Both models were implemented in Java SE8 using the Eclipse IDE. Model 1 states that with the given square footage, 1 Tesla battery is optimal, which costs USD 8,500.00. Using the given square footage and a 12 hour blackout (assumed length of the night time), Model 2 predicts the optimal battery combination to be 1 Deka battery and 1 PureStorage battery. This solution costs USD 6,851.25.

In regards to rechargeable cement batteries, research done by Zhang and Tang as well as other scientists could be monumental in the near future, if implementable. As climate change intensifies power outages caused by severe weather events will be detrimental to the population. This is where cement batteries may be useful. Concrete is a strong building material which is widely used (ScienceDaily, 2021). An experiment performed by Zhang and Tang in 2021 found that although cement batteries are not energy-dense, high volumes of cement makes up for this gap. This strongly suggests that future large buildings, such as skyscrapers, may be entirely self-sufficient when constructed with cement, if linked with a renewable energy source capable of providing enough power to the battery.

Contents

2 Newspaper Article

NOVEMBER 2021 | PROBLEM A

HIMCM GAZETTE

Team 11618

Photo from David, L. (2021, October 28). Average Cost of Solar Panels in Florida: What to Expect in 2021. EcoWatch. EcoWatch.

Amidst the global threat of climate change and unsustainable fossil fuel usage, renewable energy sources have emerged as vital technologies to satisfy future energy demands. One such technology is solar power, which captures electromagnetic radiation from the sun and converts it to electrical energy using photovoltaic solar cells. This electrical energy can be used to power any number of household appliances, or in the case of a remote home that is off-the-grid, satisfy the entire household's energy needs. There are several considerations which must be made when outfitting a property with solar capability. Each battery on the market prioritizes slightly different attributes and any number of factors can make one a better choice over another.

To help make sure homeowners are able to most effectively harness solar power, we, Team 11618, have designed an algorithmic mathematical model which can generalize to different home sizes and energy needs, all while optimizing for energy conservation and cost of the batteries. Using this model, homeowners can do their part to engage in the renewable energy movement, help to attenuate climate change, and use their money smartly. Our model of battery output simulates combinations of different batteries based on a trigonometric cycle, a power regression, and a set of checkpoints. Of the combinations which satisfy these criteria, the most cost effective one is selected. For most homes, using a Trojan battery in combination with a higher capacity lithium-ion is the optimal solution.

For residential houses, our model which considers both lead-acid batteries and lithium-ion batteries, is practical. However, for large buildings which want to invest in solar power, they may look to utilizing cement batteries, due to their low energy-density. Additional research is necessary for cement batteries to be as commercially available as residential solutions are, but we believe they could be a positive change maker in the future fight against climate change.

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3 Problem Introduction

3.1 Initial Questions and Problem Analysis

Problem A of the 2021 HiMCM math competition, titled ”Storing The Sun,” requires a model which represents a certain way to invest in an energy storage system. The energy storage system must be able to provide power for a 1600 square-foot home. Also provided in the problem is that the home is not connected to a power grid, so the energy storage system must be able to power the house independently. The energy storage system is a series of batteries storing electrical energy coming from solar panels. The problem also states the model should take different criterion of batteries into consideration: continuous power rating, instantaneous power rating, usable capacity, and round-trip efficiency. Finally, batteries have different prices, level of safety, and lifetime based on type of battery. i.e. lead-acid vs. lithium ion.

We begin the modeling process by considering questions which will affect our results and which our solution will seek to address.

1. How many people will be using energy in this home?
2. What items in the home will need energy and how much energy will they need?
3. When will people in the home use energy?
4. Are replacement fees needed in the optimal model?
5. What is the distance where connecting to the grid is too expensive?
6. What is the average 2021 house energy consumption?

3.2 Data

To formulate our models, we require data which is reflective of both solar use, and the capabilities of various battery brands. The table below shows the raw data, relating to batteries, provided in the problem or researched, as well as charge time, which we calculate using the following methodology. We use the national average of 5kW (Solstice Community Solar, 2021) as the rate of energy that the solar panels can provide to charge the batteries. We then use the voltage of a battery and its capacity to determine the amp-hours using an online calculator (Inch Calculator). Then we use the voltage, amp-hours, battery type, and solar panel wattage to calculate the charging time of a battery using a charge-time calculator (Footprinthero). Because the relationship between the charge time and solar panel input energy is linear, we then multiply the charge times by the ratio of average house size, 2301 square feet (Araj, 2021), over given house size, because the bigger the house is, the more solar panels it will have, and thus the less charge time will be for the batteries.

Battery	Cost(USD)	Battery Type	Weight (lbs)	Dimensions (inch)
Deka	368	SGLA	68	10.25 x 7.1 x 10.9
Trojan	492	FLA	118	11.7 x 6.9 x 17.6
Discover	6478	LFP	192	18.5 x 13.3 x 14.7
Electriq PP2	13000	LFP	346	27.5 x 50 x 9
Tesla Powerwall	8500	NMC	343.9	62.8 x 29.7 x 6.3
LG Chem Resu	3522.57	LFP	68	17.8 x 15.9 x 4.7
PureStorage II	6483.25	LFP	165.35	34 x 57.5 x 6.5

Table 1: Table showing 7 different batteries' cost in USD, battery type, weight in pounds, and dimensions in inches

Battery	Continuous Power Rating (kW)	Instantaneous Power Rating (kW)
Deka	0.049 (for 20 hrs.) - 0.017 (for 100 hrs.)	N/A
Trojan	0.19 (for 10 hrs.) - 0.348 (for 5 hrs.) - 0.111 (for 20 hrs.) - 0.023 (for 100 hrs.)	N/A
Discover	6.65	14.4 (for 3 s)
Electriq PP2	7.6	9 (for 1 min)
Tesla Powerwall	7	10 (for 10 s)
LG Chem Resu	3.3	3.3
PureStorage II	9	19.95 (for 3 s)

Table 2: Table showing 7 different batteries' Continuous and Instantaneous power ratings

Battery	Round-Trip Efficiency	Usable Capacity (kWh)	Charge Time
Deka	0.825*	1.18	3.3
Trojan	0.825*	2.5	3.7
Discover	0.96	7.4	4.7
Electriq PP2	0.97	10	6.0
Tesla Powerwall	0.90	13.5	7.8
LG Chem Resu	0.95	2.9	2.3
PureStorage II	0.95	10	6.2

Table 3: Table showing 7 different batteries' Round-Trip Efficiency in percentage, usable capacity in kWh, and estimated charge time.
*Round-Trip Efficiency was averaged, if applicable.

Using this data, we must now construct models which can find the optimal battery configuration for the house in question, and then can be extended to a home of any size. We must also consider new research on cement batteries and identify whether they are practical in houses, or how they could be incorporated in an off-the-grid property.

Then, we must report our solutions appropriately - in a newspaper - so that homeowners can make smart decisions when outfitting their homes with solar power. We include the newspaper article on page 3 of this paper.

4 Assumptions

1. House is located in the United States.

   The data provided in the problem is reported in USD, and the weights of the batteries are in pounds, a unit seldom used outside the United States.

2. Solar panels will provide 5 kW of power for every 2301 ft^2during daylight hours.

   This is the national average for homes that are 2301 square feet (Solstice Community Solar, 2021).

3. 1 person occupies every 971 square feet of living space.

   The average person in the United States occupies 971 square feet of home space (AEI). This is used to estimate energy consumption.

4. The house consumes an average amount of energy a day based on its square footage (also proportional to the number of people).

   Data sources in the United States report energy consumption both per person, and based on square footage; these are likely applicable to a house located in the United States, like the one we are modeling a solution for.

5. Batteries need to be able to power the house without any input of energy from solar panels for a certain number of hours chosen by the user ("blackout hours").

   On rainy or especially cloudy days, little to no solar energy will be captured, and so the battery must be able to power the home for an abnormally long period of darkness (longer than just nighttime).

6. Every day is a cycle of 12 hours of daylight, and 12 hours of darkness.

   Depending on location in the United States, as well as season/weather, the number of hours of daylight and darkness change. By making this assumption we are able to generalize to the entire contiguous United States.

7. Ceilings are 9 feet tall.

   This is the average ceiling height in the United States (Lefton, 2019).

8. All batteries except Dekas and Trojans will be modeled with a constant continuous power rating. Dekas and Trojans will be modeled with power regressions with the same rate of decay.

   The given data shows that Deka and Trojan batteries have lower continuous power ratings the longer they are used and both are lead-acid batteries. The other batteries we are considering are lithium-ion and do not lose continuous power rating over time.

9. Batteries will be fully charged at the beginning of a set of "blackout hours."

   The batteries require very little time to charge fully, so there will be enough time to charge fully between blackouts.

5 Model 1: Single Battery Brand Model

5.1 Formulation

Our first attempt to build a model results in an algorithm for ranking batteries based on multiple metrics, including amount the homeowner is willing to spend (c), weight of battery (w), battery size (v), continuous power rating (r), instantaneous power rating (i), charging time (j), round-trip efficiency (p), usable capacity (u), and number of batteries needed (n) - found by dividing the amount the homeowner is willing to spend by the cost of each of the batteries we researched. The constant terms we utilize are defined as: size of house (q), which is 14400 cubic feet, assuming 9 foot tall ceilings, needed kWh per month (m), found by multiplying the average kWh usage per person, per month, in the United States (909 kWh) times the number of people in the household (h), assuming there are 2 people living in the home because the average square feet per person in the United States is about 900. We also equate the solar power produced per month by about 20 solar panels (s) to 1800 kWh (Solstice Community Solar, 2021). In this model, accounting for rainy or cloudy days is not necessary, because the algorithm is designed in such a way that changes to solar production will not affect the ranking, because the change will be constant across the batteries being measured. **The algorithm is designed such that the better the battery is, the lower the sum x is.**

We delineate the first term of the algorithm, a_1, as being equal to 100 minus the percentage of the house that the batteries occupy, $(n \cdot v)/q$. From there, we subtract the total weight of the batteries, and divide by 100 so as to make the quantity more similar to the percentage of size. It is important to note that this factor, which we will henceforth refer to as "space," has less weight on the model than the next two terms, a_2 and a_3, because it is ultimately up to the homeowner to decide how much space they want to dedicate in their home to batteries.

$$a_1 = 100 - \frac{nv}{q} \cdot 10 - \frac{nw}{100}$$

The second term of the model, a_2, relates re-usability to the ranking system. In an optimal solution, the quotient of the monthly needed kWh divided by the capacity of the batteries would equal or be slightly less than an integer n, meaning the number of batteries used perfectly matches (or exceeds) the energy requirements of the home. By subtracting n, and then subtracting j, the charging time, this factor is calculated to reward batteries which satisfy the energy requirement of the home and low charging time, thus allowing the batteries to be reused more frequently. The difference is then multiplied by 100 in order to simultaneously benefit good batteries, where the difference of the discussed variables may be a negative - or very low positive - number, and penalize poor batteries, where the difference of the variables will be a higher number than that of a good battery.

$$a_2 = \left(\frac{m}{u} - n - j\right) \cdot 100$$

The third and final term of the model is a relative measure of power output and efficiency of the batteries. The equation begins with the kWh produced by the solar panels over the course of the month. We are treating this as a constant that ensures that the ranking algorithm *could* handle different solar inputs if the homeowner is unsure how many they want. The same reasoning is behind the inclusion of m in the next part of the equation. From m, we subtract instantaneous power rating, continuous power rating, and the round-trip efficiency (Note: round-trip efficiency is multiplied by 10 to increase the likelihood that the three metrics have the same number of place values). The difference is then multiplied by 100 to reward batteries with high efficiency and power production and to penalize poor batteries.

$$a_3 = s - (m + (-i - p \cdot 10 - r) \cdot 100)$$

The final consideration we make is how we can best combine the three aforementioned terms/criterion. We considered finding the product, but because the terms could be negative, or zero, we decided against it. For example, if two of terms were negatives of high magnitude and the third was a positive, a product would yield a high value for x, and thus a good battery would be identified mistakenly as a poor one. Thus, we find the sum of a_1, a_2, and a_3. The result is x and represents the rating index, with lower numbers being preferable and an indication of a comparatively better battery.

$$\sum_{z=1}^{3} a_z = x$$

For completeness we expand the sum:

$$x = 100 - \frac{nv}{q} \cdot 10 - \frac{nw}{100} + \left(\frac{m}{u} - n - j\right) \cdot 100 + s - (m + (-i - p \cdot 10 - r) \cdot 100)$$

5.2 Algorithm Implementation

We implement this algorithm in Java SE8, in the Eclipse IDE. The source code (Appendix 1) offers many different options for the homeowner, depending on number of people in the household and the amount they wish to spend. The code runs varying amounts spent on batteries with varying number of people in the household to find the optimal solution, based on the algorithm described above. The model only selects one brand of battery, and so combinations of battery brands are not accounted for. The existing code uses a maximum of 30,000 dollars spent on batteries (with steps of 500 dollars), and a maximum of 8 people in the household. These two maximums, as well as the step, can be changed by modulating the for-loop to the homeowner's needs and desires. A sample of some solutions produced by the model during the simulation is shown in table 4.

Number of People	Budget (USD)	Number of Batteries	Battery Brand
7	3000	6	Trojan
2	4000	1	LG
8	8000	1	PureStorage
1	19000	2	Tesla
3	28000	3	Tesla

Table 4: Table showing randomly selected output from Model 1

5.3 Model 1 Solution

To specify the model to the home in equation, we assume the 1600 square foot has 2 people living there, since national data suggests this trend (Statista, 2021). Finally, to calculate the amount of money the homeowner will likely be spending on batteries, we assume 14,000 dollars (Lane, 2019). **Based on this data, the result of Model 1 is that the homeowner should purchase 1 Tesla PowerWall+ battery.**

6 Model 2: Functional Combinations Model

Model 2 is far more sophisticated and dynamic than Model 1. Whereas the algorithm of Model 1 is built around relative indications of battery performance, Model 2 is structured around relations between the variables, and uses these relations to optimize for cost and energy requirement. Note: Model 2 uses different variable assignments than Model 1.

6.1 Continuous Power Decay

To begin, we must understand the continuous power rate decay among Deka and Trojan batteries. Let us first begin with Trojan batteries. We are given two data points: Continuous power rate is 0.19kW for 10 hours and 0.023kW for 100 hours. Online (Trojan Battery Company), we found two more continuous power rate data points: 0.348kW for 5 hours and 0.111kW for 20 hours. Thus, we have 4 data points: (5,0.348), (10,0.19), (20,0.111), (100,0.023). Using the technical computing software Mathematica, the calculator TI-nSpire CX II CAS calculator (Texas Instruments), and Desmos online graphing software, we determine a power regression model of kWt (continuous power rate of Trojan) as a function of t (time).

$$kWt[t] = 1.550335 \cdot t^{-0.9069}$$

To identify whether or not our regression model is statistically relevant, we find the coefficient of determination: R^2. This value is 0.99782..., indicating a very strong relationship between data points, and thus we know the $kWt[t]$ equation is valid.

Now, we must determine the power regression model of kW as a function of t for Deka batteries. As we are given only two data points for continuous power rate and none were found online, we must assume that the rate of continuous power rate decay is the same between Trojan and Deka. If we do not make this assumption, we have three variables and two data points, meaning no non-linear regression can be determined. This extrapolation logical because they are both lead-acid batteries. Thus, we take the rate of decay for Deka batteries of continuous power rate at -0.9069. Finally, using Mathematica, the TI-nSpire CX II CAS (Texas Instruments), and Desmos online graphing software, we determine a power regression model of kWd (continuous power rate for Deka) as a function of t (time).

$$kWd[t] = 0.630782 \cdot t^{-0.9069} + 0.00731546$$

Given the $kWd[t]$ and $kWt[t]$ equations, it is possible to determine kilowatt hours for Deka $(kWhd[t])$ and kilowatt hours for Trojan $(kWht[t])$ by taking the integral from $t = 0$ to t of $kWd[t]$ and $kWt[t]$ respectively. This is because the area under the $kW[t]$ curve(s) would be in kWh. Thus, we determine $kWhd[t]$.

$$kWhd[t] = \int_0^t kWd[t]\,dt$$

$$kWhd[t] = \int_0^t 0.630782 \cdot t^{-0.9069} + 0.00731546\, dt$$
$$kWhd[t] = 0.00731546 \cdot t + 6.77532 \cdot t^{0.0931} - 4.63802$$

Similarly, we determine $kWht[t]$.

$$kWht[t] = \int_0^t kWt[t]\, dt$$
$$kWht[t] = \int_0^t 1.550335 \cdot t^{-0.9069}\, dt$$
$$kWht[t] = 16.6459 \cdot t^{0.093136} - 11.3683$$

Finally, we must define the continuous power of all other batteries as constant, the data given in Table 2 indicates this. Therefore, the continuous kWh for Deka (d), Trojan (t), Discover (da), Electriq (e), Tesla (tp), LG (lg), and PureStorage (ps) are as follows:

$$kWhd[t] = 0.00731546 \cdot t + 6.77532 \cdot t^{0.0931} - 4.63802$$
$$kWht[t] = 16.6459 \cdot t^{0.093136} - 11.3683$$
$$kWhda[t] = 6.65 \cdot t$$
$$kWhe[t] = 7.6 \cdot t$$
$$kWhtp[t] = 7 \cdot t$$
$$kWhlg[t] = 3.3 \cdot t$$
$$kWhps[t] = 9 \cdot t$$

These functions will be employed in Model 2, the code for which can be found in Appendices 2 and 3..

6.2 Necessary Energy Output

For an accurate solution, we must also model out energy usage as a function of time, because energy draw is not constant at all hours of the day (PJM) For ease of calculation, we take $t = 0$h to mean 6:00AM (Sunrise). To begin, we must use our assumption that the home used 7450.64kWh per month (The US average, scaled for home size), which can be translated into kWh/hour by dividing by the number of hours in the average month, which is known to be 8766. $\frac{7450.64}{8766} = 0.849947524527$ kWh per hour is needed, which means our battery must be able to output 0.85kW consistently. However, the amount of energy needed by our household can be more accurately modeled by a trigonometric cosine function, which allows us to account for differences between day and night time usage. Let us first establish the template for our cosine model as follows:

$$n[t] = a \cdot \cos(b \cdot (t - c)) + d$$

Now we must begin determining the coefficients a, b, c, and d. Allow us to begin with b. Since we are modeling energy need as a repeating 24 hour cycle, we know the period must be 24. The period of a function may be expressed as $\frac{2 \cdot \pi}{b}$ where b is the coefficient of x. Therefore, we know that $b = 0.261799$. Now we find horizontal shift c. We need the minimum of the curve at 0 hours and at 24 hours (PJM), so we shift the cosine curve 12 hours to the right, which gave a c value of 12. To find d, we take the total energy the house needs for a year: 7450.674 kWh. This means that over the course of 8766 hours (one year) the house uses 7450.674 kWh. Thus, it needs 0.84995 kWh per hour (on average). This gives us our midline, and our vertical shift d as 0.84995. Now, to find the amplitude (a), we analyzed data regarding the power draw for a power grid during an average day, in which the high was 450 million kWh and the low was 350 million kWh. We use this as a ratio. (High)/(Low) $= \frac{450000000}{350000000}$. This means that $\frac{d+a}{d-a} = 9/7$. Solving this equation for a, knowing that $d = 0.84995$, we find the final coefficient, a, to be 0.10624. We delineate energy need as a function of b as follows:

$$n[t] = 0.106244 \cdot \cos(0.2617993 \cdot (t - 12)) + 0.84995$$

6.3 Modeling System Energy

Model 2 simulates every single combination of batteries and chooses the most cost-effective whilst satisfying energy needs, and even meeting the needs of a rainy day scenario, where we assume total blackout for a given number of hours. At any point during the day, the total usable energy stored must be greater than the required energy from the home at that time. In a 24 hour period, if time $t = 0$ represents the start of the day cycle, and $t = 24$ represent the next start of the day cycle and the end of the previous night cycle, $t = 12$ represents the switch over between light to darkness. The time at which the sun sets and the panels stop producing energy is the same (PJM) to the time at which the house is requiring the most energy: $t = 12$, which is 6:00PM. The cosine function represents the house's needs, while a piecewise function represents the amount of energy stored during the 24 hour cycle. Now, we must find this piecewise function. First, we must state that the stored energy in the batteries at the end of the night must be the same as the stored energy at the start of the day. This is because if it were to decrease, as we cycle days it would go to 0, which means the house would lose power. Research yields that the rate of energy being produced from solar panels (on average) in the US during the course of the average day is 5 kW, which is more than the required energy from the house modeled by the cosine function. We break up the day cycle into 4 points. t_1 is the starting power stored at the beginning of the day, when the sun rose, and is equal to power stored at t_4 (The end of day 1 is the beginning of day 2). t_2 is the point when the battery system reaches maximum energy capacity. $t_3 = 12$ is when the sun goes down, so energy stored begins decreasing. Finally, t_4 is the very end of the day. If this point is above the required energy at that time, the battery has lasted the night. Figure 1 shows this because the section t_2 to t_3 is a horizontal line at the battery's (or combination of batteries') maximum storage.

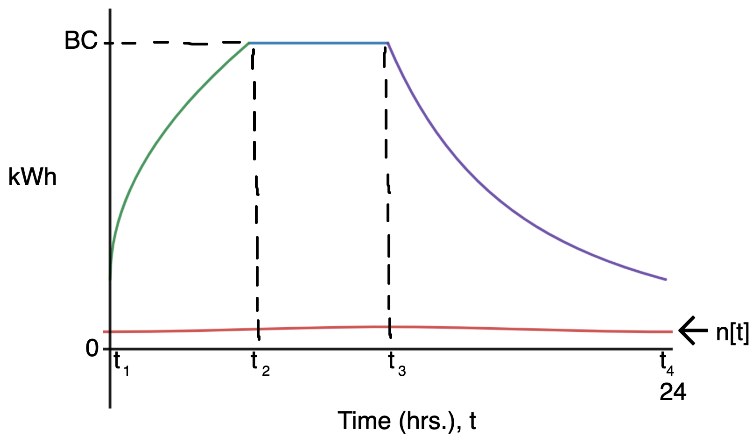

Figure 1: Graph of kWh[t], not drawn to scale

The section t_1 to t_2 is the nonlinear curve representing the amount of energy in the battery, as it increases from its sub-maximum amount which it dropped to throughout the night. It is both emitting energy represented by the cosine curve, as well as being charged at a constant rate. This curve of kWh[t] for t_1 to t_2 can be defined as follows, where r is the rate of charge.

$$kWh[t] = kWh[t_1] + r \cdot t - \int_0^t n[t]dt$$

$$kWh[t] = kWh[t_1] + 5 \cdot t - \int_0^t (0.106244 \cdot \cos(0.2617993 \cdot (t - 12)) + 0.84995)dt$$

Now, we find the function of kWh[t] from t_2 to t_3, which is a constant defined as the total battery capacity, BC.

$$kWh[t] = BC$$

Now we must find the curve from t_3 to t_4. This is represented by battery capacity decreasing by the integral of n[t] (The cosine function representing energy need) over 12 hours from $t_3 = 12$ to $t_4 = 24$.

$$kWh[t] = kWh[t_3] - \int_{12}^{24} n[t]dt$$

$$kWh[t] = kWh[t_3] - \int_{12}^{24} (0.106244 \cdot \cos(0.2617993 \cdot (t - 12)) + 0.84995)dt$$

From which we can find the kWh[t_4] as a function of kWh[t_3]

$$kWh[t_4] = kWh[t_3] - 10.2$$

Finally, we express kWh[t] as a piecewise function.

$$kWh[t] = \begin{cases} kWh[t_1] + 5 \cdot t - \int_0^t (0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt & t_1 \le t \le t_2 \\ BC & t_2 \le t \le t_3 \\ kWh[t_3] - \int_{12}^t (0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt & t_3 \le t \le t_4 \end{cases}$$

We can now re-write the piecewise function, replacing t_1 with 0, t_3 with 12, and t_4 with 24. Notice that kWh[t] for $t_2 \le t \le t_3$ is simply BC, which is a known property of any group of batteries, so our goal is to express the variables in terms of BC. We therefore replace kWh[t_2] and kWh[t_3] with BC.

$$kWh[t] = \begin{cases} kWh[0] + 5 \cdot t - \int_0^t (0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt & 0 \le t \le t_2 \\ BC & t_2 \le t \le 12 \\ BC - \int_{12}^t (0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt & 12 \le t \le 24 \end{cases}$$

Now, we can find $kWh[t_1]$ (the same as $kWh[t_4]$) using the third equation in terms of BC, by substituting in $t = 24$. We find that $kWh[t_1] = kWh[t_4] = BC - 10.2$.

Then we find t_2: the time during the day at which the battery charges back to its usable capacity after losing energy in the night. We find t_2 by substituting it into the first equation of the piecewise function, and equating it equal to BC.

$$(BC - 10.2) + 5 \cdot t_2 - \int_0^{t_2} (n[t])dt = BC$$

Now, if we solve this function we find $t_2 = 2.40018$. The reason why BC does not impact the value of t_2 is because the Battery Capacity will only vertically shift the graph of $kWh[t]$. Thus, we may re-write our piecewise function, replacing t_2 with 2.4.

$$kWh[t] = \begin{cases} BC - 10.2 + 5 \cdot t - \int_0^t (0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt & 0 \le t \le 2.4 \\ BC & 2.4 \le t \le 12 \\ BC - \int_{12}^t (0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt & 12 \le t \le 24 \end{cases}$$

This piecewise function is an accurate and precise representation of the kWh in the energy storage system as a function of time. It also considers the BC (Battery Capacity) of the system.

6.4 Functions of the Model

Now we are ready to discuss the functions of the model. The model will generate every possible combination of batteries that satisfies the daily energy needs, test them, and then finally select the cheapest combination that passes all tests. We will incorporate three tests. In order to prove that the combination of batteries is able to provide for a home during a cyclic period of 24 hours, the battery capacity for the night (in kWh) must be greater than or equal to the accumulation (integral) of the house's needs every hour during the night. This gives us our first test for the model:

$$BC \ge \int_{12}^{24} (0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt$$

If this first test is passed, the combination is checked against the second test. For our second check, we focus on the charge time it takes for a combination of batteries. To make sure that the batteries have time to charge during the day, we check that the total charge time of all the batteries is less than the 12 hours which they can be charged (daylight). The second check ensures that the sum of the numbers of each type of battery in a certain combination multiplied by its corresponding charge time, is less than 12 hours.

Now, we progress to our third and final check. For our third check, we ensure that even during the day the batteries are able to provide the energy that the house requires for a certain amount of "blackout" time. We call this our rainy-day check. We assumed that the beginning of the blackout occurs when all of the batteries are fully charged: anywhere from t_2 to t_3 in Figure 1. In order to create a check that returns a boolean value, we notice that the total energy stored must be greater than the energy required by the house (the cosine function). This check takes in input for required blackout (BO) time, because different families may have different needs. For reference, a blackout time of 12 hours is simply a regular night. We delineate our third and final check:

$$BC \geq \int_0^{24}(0.106244 \cdot \cos(0.2617993 \cdot (t-12)) + 0.84995)dt) \cdot BO/24$$

Now that we have can find all battery combinations that pass the three checks, we must select an optimal combination. We do this by finding the combination with the minimum price.

6.5 Model 2 Results

Blackout (hrs.)	**12 (Normal)**	14	16	18	20
Deka(s)	**1**	0	1	0	0
Trojan(s)	**0**	1	0	1	0
Discover(s)	**0**	0	0	0	1
Electriq(s)	**0**	0	0	0	0
Tesla(s)	**0**	0	1	1	0
LG(s)	**0**	0	0	0	0
PurStorage(s)	**1**	1	0	0	1
Cost (USD)	**6,851.25**	6,975.25	8,868.00	8,992.00	12,961.25

Table 5: Results from Model 2 at a certain number of "repeating blackout" hours for a house size of 1600 ft^2. What this means is that we can expect this blackout to repeat in the future, and require the necessary energy stored at all times.

Note that blackout hours only go up to 20 for a 1600ft^2 house. This is because at some point, the number of batteries to sustain a blackout will not be able to fully charge over a 12 hour day. However, we adjusted the model slightly, to make a "Emergency Blackout Model" in which the batteries used to sustain the blackout will not be used in the regular 12 hour day/night cycle, and therefore do not need to be charged during the 12 hour day.

Our solution to this problem with a regular day night cycle of 12 hours each is 1 Deka and 1 PurStorage Battery.

Emergency Blackout (hrs.)	21	22	23	24
Deka(s)	1	0	0	1
Trojan(s)	3	4	4	4
Discover(s)	0	0	0	0
Electriq(s)	0	0	0	0
Tesla(s)	0	0	0	0
LG(s)	0	0	0	0
PureStorage(s)	1	1	1	1
Cost (USD)	8,327.25	8,451.25	8,451.25	8,819.25

Table 6: Results from Model 2 with emergency blackout hours. Notice the optimal price is lower than in table 5. This is because these batteries are no longer constrained by charge time, and therefore cheaper batteries may be purchased for one time use (They are purchased fully charged, used once for the emergency blackout, and not charged again, unless by another unconsidered source).

6.6 Generalizing Model 2

In order to generalize model 2, we express household annual energy draw (h) as a function of size (S) in ft^2. We need to know US median square feet per occupant (dpt), as well as US median power per person (ppp).

$$h = \frac{S}{dpt} \cdot ppp \cdot 12$$

$$h = S \cdot 11.2338$$

Now, we must express the coefficients of $n[t]$, the function of energy need, in terms of h.

$$n[t] = a \cdot \cos(b \cdot (t - c)) + d$$

Allow us to begin with b. We know that b, the period of the function, is independent of home size, because the cycle remains 24 hours. Therefore we may use the fact that the period of a function may be expressed as $\frac{2 \cdot \pi}{b}$ where b is the coefficient of x. Therefore, we know that $b = 0.261799$. Now, allow us to find c. Similar to b, c is independent of house size. The peak of our cosine function when $c = 0$ is always at $t = 0$. Regardless of house size, the peak energy usage happens at 6:00PM ($t = 12$), so the cosine curve must be shifted right by 12. Therefore c is always 12. Now, we will determine the dependent coefficients. The coefficient d, the vertical shift, representative of the average hourly energy usage, can be determined as follows:

$$d = \frac{h}{hoursperyear}$$

$$d = \frac{h}{8766}$$

Thus, d is expressed in terms of h. The coefficient a can be determined by the following equation.

$$\frac{d+a}{d-a} = \frac{9}{7}$$

$$a = \frac{d}{8}$$

$$a = \frac{h}{70128}$$

And thus, a, b, c, and d are expressed in terms of h. Now we can express $n[t]$ in terms of h.

$$n[t] = a \cdot \cos(b \cdot (t - c)) + d$$

$$n[t] = \frac{h}{70128} \cdot \cos(0.261799 \cdot (t - 12)) + \frac{h}{8766}$$

To generalize model 2 to take house size in ft^2 as an input, we must write n[t] in terms of h in our model code. Below is an example of results with a different square footage.

Blackout (hrs.)	**12 (Normal)**	14	16	18	20
Deka(s)	**1**	0	0	0	0
Trojan(s)	**3**	4	3	2	1
Discover(s)	**0**	0	0	0	0
Electriq(s)	**0**	0	0	0	0
Tesla(s)	**0**	0	2	1	3
LG(s)	**0**	0	0	0	0
PurStorage(s)	**2**	2	0	2	0
Cost (USD)	**14,810.50**	14,934.50	18,476.00	22,450.50	25,992.00

Table 7: Results From Generalized Model 2 at a certain number of “repeating blackout” hours for a house size of 4000 ft^2

7 Cement Batteries

7.1 Overview

Concrete in construction is mainly found in public structures, such as large buildings, bridges, roads, street lamps, and much more (Knight, 2018). Modern research has indicated a possibility of incorporating a cathode and anode into the cement to produce concrete capable of receiving and outputting energy, and most importantly, store energy (Zhang and Tang, 2021). This may be applied to all future construction projects involving high amounts of cement, as the world is moving towards a cleaner future which means clean and renewable sources of energy. Buildings currently consume 1/3 of global energy and generate 40% of all carbon emissions (Pavel, 2021). Solar energy or other electricity-generating methods may be stored in the concrete of a building, reducing carbon emissions. Power can be stored within the concrete and later used when needed, such as during the winter months (Pinkstone, 2018). For example, cement batteries that are part of a skyscraper will be able to provide electricity, such as lighting, when needed (Pavel, 2021). Other uses could be to store energy in the concrete of monitoring systems, such as bridges, to help warn potential of structural failure (Pavel, 2021). Cement batteries have the potential to end a majority of energy problems (Pinkstone, 2018).

7.2 Advantages and Disadvantages

Pros:

1. Assuming volume of concrete used to construct building is sufficient, building becomes self-sufficient (Pavel, 2021).
2. Cement batteries are built into the structure so inside space will not be needed to store them (Zhang and Tang, 2021).
3. Batteries within the walls can detect signs of weakening infrastructure (Zhang and Tang, 2021).
4. Cement batteries also have the potential to provide power to small appliances individually (Knight, 2018).
5. Cement is a common and useful building material, so incorporating a battery into the material would be beneficial (ScienceDaily, 2021).
6. Cement batteries are cheap to implement (Knight, 2018).

Cons:

1. The process of implementing large-scale cement batteries into structures of large buildings emits high concentrations of carbon.
2. The energy density of cement batteries is about 10 times less than conventional batteries (Delbert, 2021).
3. Low energy density means need for high volume, so cement batteries will not be as feasible for smaller structures, such as small homes.
4. Concrete blocks store energy far less efficiently than conventional batteries.
5. Charge cycle of cement batteries is longer than other batteries, and once the cement battery dies, it is difficult to replace.
6. Cement batteries have a poor performance level compared to typical solar storage systems (Zhang and Tang, 2021).

7.3 Uses in Remote Home

In most circumstances, cement batteries will not be capable of storing enough energy. This is because of a large issue of cement batteries mentioned in the previous section: a very low energy density when compared to weight. The reason cement batteries have the opportunity to be implemented in construction is because of the sheer amount of cement required to build these structures. When the total volume of cement is not great enough, there will not be enough energy stored within the walls. Fortunately, small amounts of concrete may still be useful. Studies have shown that smaller cement battery proportions may be used to power individual appliances, such as a television or charging outlet. If the house is primarily constructed using cement, then fewer "typical" batteries need to be bought/utilized, as more cement used means more energy capacity. There could be the possibility of cement batteries capable of powering heating/AC or lights constantly. In summation, the greater the volume of cement is used, the greater the amount of energy will be stored within the walls to power appliances used every day. This means they are generally a poor option in small or medium sized houses.

7.4 Uses in Any Home

On the other hand, concrete batteries may be effectively incorporated into larger houses or apartment buildings, as long as concrete is used to build the external structure of the building, as the energy density of concrete is lower than typical solar storage batteries (Pinkstone, 2018). It would be illogical to attempt to store enough energy to power a large office building using a very small volume of cement infused with battery components. In the future, skyscrapers or office buildings may be self-sufficient on purely cement batteries and solar panels. Street lights could also solely rely on solar energy.

7.5 Information Needed to Model Cement Batteries

1. Cost of battery-cement and traditional building materials, such as wood or brick used to construct the house. Since the solar storage system is incorporated in the building material of the house, the cost of battery cement to the material used in constructing the house needs to be mentioned to get an accurate representation of which storage system is the most cost-efficient yet satisfying criterion.

2. A cement battery's power ratings/efficiency. This information is needed to determine the ability of a cement battery to satisfy the criterion, such as continuous power rating to see how well the cement battery is able to supply energy during the night or a "blackout."

3. A cement battery's ratio of volume and output of energy. This information is also needed to determine if the cement battery is capable of satisfying the criterion

4. The volume of concrete used in building the house. Relating to the information needed above, the volume of concrete is needed to calculate the quantity of energy the entire cement battery is capable of producing. This is important because the amount of electricity the cement battery is capable of outputting into the house needs to exceed the total amount of energy the house requires to function properly.

5. Charge time of cement battery. This is important because the model incorporates charge time of different batteries to compare them, effectively determining the optimal combination of batteries.

8 Conclusion and Sensitivity Analysis

In this paper we developed two models - one simpler, Model 1, and one more sophisticated, Model 2 - in order to determine the best battery storage solution for a home using solar energy with no access to a power grid. The models are based off of multiple battery characteristics including continuous power rating, instantaneous power rating, round-trip-efficiency, usable capacity, and cost. We also account for home size and blackout time. The models differ in variable assignment and considered factors, as well as capability with regards to battery combinations, but both consider a myriad of factors.

Models were targeted towards a 1600 square foot home that is off the grid, and were then generalized to include several other determining factors (e.g. household size, home size, budget). Model 2 benefited from the ability to determine combinations of battery brands, as well as the ability to test whether solutions were feasible, based on energy use. Model 1 did not assess feasibility (though the generated solutions were generally feasible when realistic sums of money were spent), and did not consider combinations. Model 1 is better applied to a very small property, perhaps a tiny house or small mobile home, where combinations of battery brands will likely be difficult due to space concerns. Model 2 is better overall, and useful for all sizes of homes due to its optimization of cost.

In the future we would aim to present our solution in a website format so that homeowners using solar energy to power their homes could easily provide inputs, such as blackout time they want to be prepared for, as well as the size of their home. With a more specific location in the United States we predict the accuracy of our solution would improve, because blackout hours could be more reliably considered depending on regional weather. Additionally, if the homeowner owns an electric vehicle, we may need to utilize additional battery power to charge the car. The final factor which could impact rationality of the model is the material the house is made of, and how well the home insulates heating and cooling.

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10 Appendix

10.1 Model 1 Code

```
public class StoringTheSun {}
public static void main(String[] args) {
double c;
double h;
for (c=500; c<=30000; c=c+500) {
for(h=1;h<=8;h++) {
// Discover
int n1 = (int) (c / 6478);
double w = 192;
double r = 6.65;
double i = 14.4;
double p = .95;
double u = 7.4;
double v = 3616.935;
double s = 1800;
double q = 14400;
double j = 3.3*20709/q;
double k = 909;
double m = k * h;
double a1 = 100 - n1 * (v) / q * 10 - n1 * (w) / 100;
double a2 = ((m / u - n1)-j) * 100;
double a3 = s - (m + (-i + p*10 - r) * 100);
double xDiscover;
if (n1 <= 0)
xDiscover = 100000000;
else
xDiscover = a1 + a2 + a3;
// Deka
int n2 = (int) c / 368;
w = 68;
r = 0.017;
i = 0;
p = .825;
u = 1.18;
v = 793.2475;
```

```
j =2.3*20709/q;
a1 = 100 - n2 * (v) / q * 10 - n2 * (w) / 100;
a2 = ((m / u - n2)-j) * 100;
a3 = s - (m + (-i + p*10 - r) * 100);
double xDeka;
if (n2 <= 0)
xDeka = 100000000;
else
xDeka = a1 + a2 + a3;
// Trojan
int n3 = (int) c / 492;
w = 118;
r = 0.023;
i = 0;
p = .825;
u = 2.5;
v = 1420.848;
j =2.6*20709/q;
a1 = 100 - n3 * (v) / q * 10 - n3 * (w) / 100;
a2 = ((m / u - n3)-j) * 100;
a3 = s - (m + (-i + p*10 - r) * 100);
double xTrojan;
if (n3 <= 0)
xTrojan = 100000000;
else
xTrojan = a1 + a2 + a3;
// Electriq
int n4 = (int) c / 13000;
w = 346;
r = 7.6;
i = 9;
p = .966;
u = 10;
v = 12375;
j =4.2*20709/q;
a1 = 100 - n4 * (v) / q * 10 - n4 * (w) / 100;
a2 = ((m / u - n4)-j) * 100;
a3 = s - (m + (-i + p*10 - r) * 100);
double xElectriq;
if (n4 <= 0)
xElectriq = 100000000;
else
xElectriq = a1 + a2 + a3;
// Tesla
int n5 = (int) c / 8500;
w = 343.9;
v = 11750.508;
r = 7;
i = 10;
p = .9;
u = 13.5;
j =5.4*20709/q;
```

```
a1 = 100 - n5 * (v) / q * 10 - n5 * (w) / 100;
a2 = ((m / u - n5)-j) * 100;
a3 = s - (m + (-i + p*10 - r) * 100);
double xTesla;
if (n5 <= 0)
xTesla = 100000000;
else
xTesla = a1 + a2 + a3;
// LG
int n6 = (int) c / 3523;
w = 68;
v = 1330.19;
r = 3.3;
p = .95;
u = 2.9;
i = 3.3;
j =1.6*20709/q;
a1 = 100 - n6 * (v) / q * 10 - n6 * (w) / 100;
a2 = ((m / u - n6)-j) * 100;
a3 = s - (m + (-i + p*10 - r) * 100);
double xLG;
if (n6 <= 0)
xLG = 100000000;
else
xLG = a1 + a2 + a3;
// PureStorage
int n7 = (int) c / 6483;
w = 165.25;
v = 12707.5;
r = 9;
p = .95;
i = 19.95;
u = 10;
j =4.3*20709/q;
a1 = 100 - n7 * (v) / q * 10 - n7 * (w) / 100;
a2 = ((m / u - n7)-j) * 100;
a3 = s - (m + (-i + p*10 - r) * 100);
double xPS;
if (n7 <= 0)
xPS = 100000000;
else
xPS = a1 + a2 + a3;
if (xElectriq > xDiscover && xDeka > xDiscover && xTrojan > xDiscover && xTesla >
xDiscover && xLG
> xDiscover && xPS > xDiscover)
System.out.println("If you have " + h + " people in the household, and spend " + c + ",
" + n1 + " Discover is the best choice.");
else if (xDiscover > xElectriq && xDeka > xElectriq
&& xTrojan > xElectriq && xTesla > xElectriq && xLG > xElectriq
&& xPS > xElectriq)
System.out.println("If you have " + h + " people in the household, and spend " + c + ",
" +n4 + " Electriq is the best choice.");
```

```
else if (xDeka > xTrojan && xElectriq > xTrojan
&& xDiscover > xTrojan && xTesla > xTrojan && xLG
> xTrojan && xPS > xTrojan)
System.out.println("If you have " + h + " people in the household, and spend " + c + ",
" +n3 + " Trojan is the best choice.");
else if (xDeka > xTesla && xElectriq > xTesla && xDiscover
> xTesla && xTrojan > xTesla && xLG > xTesla && xPS > xTesla)
System.out.println("If you have " + h + " people in the household, and spend " + c + ",
" +n5 + " Tesla is the best choice.");
else if (xDeka > xLG && xElectriq > xLG && xDiscover
> xLG && xTrojan > xLG && xTesla > xLG && xPS > xLG)
System.out.println("If you have " + h + " people in the household, and spend " + c + ",
" +n6 + " LG is the best choice.");
else if (xDeka > xPS && xElectriq > xPS && xDiscover
> xPS && xTrojan > xPS && xTesla > xPS && xLG > xPS)
System.out.println("If you have " + h + " people in the household, and spend " + c + ",
" +n7 + " PureStorage is the best choice.");
else
System.out.println("If you have " + h + " people in the household, and spend " + c + ",
" +n2 + " Deka is the best choice.");}}}}
```

10.2 Model 2 Battery Class

```
public class Battery {
public double price;
public double weight;
public double volume;
public double cpr;
public double ipr;
public double rte;
public double usableCap;
public double chargeTime;
public Battery (double priceA, double weightA, double volumeA, double cprA, double iprA,
double rteA, double usableCapA, double chargeTimeA) {
price = priceA;
weight = weightA;
volume = volumeA;
cpr = cprA;
ipr = iprA;
rte = rteA;
usableCap = usableCapA;
chargeTime = chargeTimeA;}
```

10.3 Model 2 Implemented Code

```
public class FinalStoringTheSunModel {
public static final double size = 1600;
public static final double householdEnergy = 10715*size/2301;
public static final double hblackout = 12;
public static void main(String[] args) {
//Batteries
//Deka Solar 8GCC2 6V 198
```

```
Battery deka = new Battery(368.0, 68.0, 793.2475, 0.017, 0, 0.825, 1.18, 2.2*2301/size);
/*double dekaPrice = 368.0;
double dekaWeight = 68.0;
double dekaVolume = 793.2475;
double dekaCPR = 0.017;
double dekaIPR = 0;
double dekaRTE = 82.5;
double dekaUsableCap = 1.18;*/
//Trojan L-16 -SPRE 6V 415
Battery trojan = new Battery(492.0, 118.0, 1420.848, 0.023, 0, 0.825, 2.5,
    2.6*2301/size);
/*double trojanPrice = 492.0;
double trojanWeight = 118.0;
double trojanVolume = 1420.848;
double trojanCPR = 0.023;
double trojanIPR = 0;
double trojanRTE = 82.5;
double trojanUsableCap = 2.5;*/
//Discover AES 7.4 kWh
Battery discover = new Battery(6478.0, 192.0, 3616.935, 6.65, 14.4, 0.95, 7.4,
    3.3*2301/size);
/*double discoverPrice = 6478.0;
double discoverWeight = 192;
double discoverVolume = 3616.935;
double discoverCPR = 6.65;
double discoverIPR = 14.4;
double discoverRTE = 95;
double discoverUsableCap = 7.4;*/
//Electriq PowerPod 2
Battery electriq = new Battery(13000.0, 346.0, 12375.0, 7.6, 9.0, 0.966, 10.0,
    4.2*2301/size);
/*double electriqPrice = 13000.0;
double electriqWeight = 346.0;
double electriqVolume = 12375;
double electriqCPR = 7.6;
double electriqIPR = 9;
double electriqRTE = 96.6;
double electriqUsableCap = 10;*/
//Tesla Powerwall+
Battery tesla = new Battery(8500.0, 343.9, 11750.508, 7.0, 10.0, 0.900, 13.5,
    5.4*2301/size);
/*double teslaPrice = 8500.0;
double teslaWeight = 343.9;
double teslaVolume = 11750.508;
double teslaCPR = 7;
double teslaIPR = 10;
double teslaRTE = 90.0;
double teslaUsableCap = 13.5;*/
/*System.out.println(combs1(deka));
System.out.println(combs1(trojan));
System.out.println(combs1(discover));
System.out.println(combs1(electriq));
```

```
System.out.println(combs1(tesla));*/
Battery LifeG = new Battery(3522.57, 68, 1330.19, 3.3, 3.3, 0.950, 2.9,
    1.6*2301/size);
//Battery tesla = new Battery(8500.0, 343.9, 11750.508, 7.0, 10.0, 90.0, 13.5, 5.4, 3);
Battery PureStorage = new Battery(6483.25, 165.35, 12707.5, 9.0, 19.95, 0.950, 10.0,
    4.3*2301/size);
double cheapest = 1.8*Math.pow(10, 308);
int cheapd = 0;
int cheapt = 0;
int cheapdi =0;
int cheape= 0;
int cheapte =0;
int cheaplg = 0;
int cheapPS = 0;
int cheapd2 = 0;
int cheapt2 = 0;
int cheapdi2 =0;
int cheape2= 0;
int cheapte2 =0;
int cheaplg2 = 0;
int cheapPS2 = 0;
for(int d = 0; d<=howManyDeka(hblackout); d++) {
for(int t = 0; t<=howManyTrojan(hblackout); t++) {
for(int di = 0; di<=combs1(discover); di++) {
for(int e = 0; e<=combs1(electriq); e++) {
for(int te = 0; te<=combs1(tesla); te++) {
for(int lg = 0; lg<=combs1(LifeG); lg++) {
for(int PS=0; PS<=combs1(PureStorage); PS++) {
if(blackOut(d, t, di, e, te, lg, PS, deka, trojan, discover, electriq, tesla, LifeG,
    PureStorage)==true) {
for(int d2 = 0; d2<=d; d2++) {
for(int t2 = 0; t2<=t; t2++) {
for(int di2 = 0; di2<=di; di2++) {
for(int e2 = 0; e2<=e; e2++) {
for(int te2 = 0; te2<=te; te2++) {
for(int lg2 = 0; lg2<=lg; lg2++) {
for(int PS2=0; PS2<=PS; PS2++) { if(workEmission(d2, t2, di2, e2, te2, lg2, PS2, deka,
trojan, discover, electriq, tesla, LifeG, PureStorage)&&chargeWork(d2, t2, di2, e2, te2,
lg2, PS2, deka,
trojan, discover, electriq, tesla, LifeG, PureStorage)) {
if(cost(d, t, di, e, te, lg, PS)<cheapest) {
cheapest = cost(d, t, di, e, te, lg, PS); cheapd = d; cheapt = t; cheapdi =di; cheape= e;
cheapte =te; cheaplg = lg; cheapPS = PS; cheapd2 = d2; cheapt2 = t2;
  cheapdi2=di2 cheape2= e2; cheapte2 =te2; cheaplg2 = lg2; cheapPS2 = PS2;
  }}}}}}}}}}}}}}}}
System.out.println("Batteries which will be used in the emergency blackout: ");
System.out.println(cheapd);
System.out.println(cheapt);
System.out.println(cheapdi);
System.out.println(cheape);
System.out.println(cheapte);
System.out.println(cheaplg);
```

```
System.out.println(cheapPS);
System.out.println();
System.out.println("Batteries which will be used in the regular 12 day/night cycle: ");
System.out.println(cheapd2);
System.out.println(cheapt2);
System.out.println(cheapdi2);
System.out.println(cheape2);
System.out.println(cheapte2);
System.out.println(cheaplg2);
System.out.println(cheapPS2);
System.out.println();
System.out.println("Cheapest Price:");
System.out.println(cheapest);}
public static int combs1 (Battery a) {
boolean works = false;
int count = 0;
while(works==false) {
if((a.usableCap*count/hblackout)>=((householdEnergy)/(8766)+householdEnergy/(8*8766))) {
works = true;
} else {
count++;}}
return count;}
public static int howManyDeka (double a) {
boolean works = false;
int count = 0;
double time2 = 0;
while(works==false) {
for(double time1 = 0; time1<a; time1+=0.1) {
if((dekaOut(time1))<=NecessaryOut(time2)) {
count++;
time1=0;}
if(time2>=a) {
works=true;
time1=a+6;}
time2+=0.1;}}
return count;}
public static int howManyTrojan (double a) {
boolean works = false;
int count = 0;
double time2 = 0;
while(works==false) {
for(double time1 = 0; time1<a; time1+=0.1) {
if((trojanOut(time1))<=NecessaryOut(time2)) {
count++;
time1=0;}
if(time2>=a) {
works=true;
time1=a+6;}
time2+=0.1;}}
return count;}
public static double cost (int deka, int trojan, int discover, int electriq, int tesla,
int LifeG, int PureStorage) {
```

```
return 368.0 * deka + 492.0 * trojan + 6478.0 * discover + 13000.0 * electriq + 8500.0 *
tesla + 3522.57 * LifeG + 6483.25 * PureStorage;}
public static boolean chargeWork (int deka, int trojan, int discover, int electriq,
int tesla, int LifeG, int PureStorage, Battery d, Battery t, Battery di, Battery e,
Battery te, Battery lg, Battery PS) {
if(((double)deka * d.chargeTime+(double)trojan * t.chargeTime+(double)discover *
di.chargeTime+(double)electriq * e.chargeTime+(double)tesla * te.chargeTime+(double)
LifeG * lg.chargeTime+(double)PureStorage * PS.chargeTime)<=12) {
return true;}
return false;}
public static boolean blackOut(int deka, int trojan, int discover, int electriq,
int tesla, int LifeG, int PureStorage, Battery d, Battery t, Battery di, Battery e,
Battery te, Battery lg, Battery PS) {
double batCap = deka*d.usableCap+trojan*t.usableCap+discover*di.usableCap+electriq*
e.usableCap+tesla*te.usableCap+LifeG*
lg.usableCap+PureStorage*PS.usableCap;
double needs = 0.0013689256*householdEnergy*hblackout/12;
if(batCap>needs) {
return true;}
return false;}
public static double dekaOut (double time) {
return 0.630782 * Math.pow(time, -0.9069)+0.00731546;}
public static double trojanOut (double time) {
return 1.550335* Math.pow(time, -0.9069);}
public static double NecessaryOut (double time) {
double a=householdEnergy/(8*8766);
double b=Math.PI/(24);
double c=householdEnergy/(8766);
return a*Math.cos(b*(time-12))+c;}
public static boolean workEmission (int deka, int trojan, int discover, int electriq,
int tesla, int LifeG, int PureStorage, Battery d, Battery t, Battery di, Battery e,
Battery te, Battery lg, Battery PS) {
double batCap = deka*d.usableCap+trojan*t.usableCap+discover*di.usableCap+electriq*
e.usableCap+tesla*te.usableCap+LifeG*lg.usableCap+PureStorage*PS.usableCap;
/*double valOft1 = batCap-0.0013689256*householdEnergy;
double t2 = 1.71342;
double extraEnergy = 11.287*batCap-2.91262-0.0013689256*householdEnergy;*/
double RTE = ((deka * d.rte * d.usableCap)+(trojan*t.rte*t.usableCap)+(discover*
di.rte*di.usableCap)+(electriq*e.rte*e.usableCap)+(tesla*te.rte*te.usableCap)+
(LifeG*lg.rte*lg.usableCap)+(PureStorage*PS.rte*PS.usableCap))/((batCap));
/*if((extraEnergy*RTE)>=0.0013689256*
householdEnergy&&batCap>=0.0013689256*householdEnergy) {
return true;
}*/
if(RTE*batCap>0.0013689256*householdEnergy) {
return true;
}
return false;}}
```