STEM I Literature Review

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Introduction

Tournaments are a way of matching up teams against each other, to determine which team is the most skilled in a particular activity. The main goal of a tournament is to reward higher skill with a higher chance of victory. Some sports tournaments achieve this goal through seeding the tournament, which involves ranking the players in terms of skill and deciding matchups based on that (Baumann, Matheson & Howe, 2010). For example, the standard method of seeding consists of matching the teams up so that the sums of the team numbers are constant (Schwenk, 2000). Another method ensures fairness by reseeding multiple times throughout the tournament to reevaluate matchups (Baumann et al., 2010). The tournaments that this paper focuses on are horseshoe tournaments. While some more official horseshoe tournaments use ringer percentage, a form of skill level that allows organizers to rank people into seeds, most horseshoe tournaments lack seeding or reseeding (NHPA, 2019). Horseshoe tournaments are set up by deciding teams of two, and these teams will compete until an objective, which varies between types of tournaments, is completed. For example, double elimination requires teams to lose twice to be eliminated, and the objective is to be last team that is not eliminated . The organizer of the tournament determines these teams by a random draw, which does not always lead to the best results. For example, two people that have the most amount of skill can become partners. While the tournament does favor them in terms of skill, it may not favor the 3rd best player who is also very skilled but became partners with someone of low skill level. These types of matchups would not be considered fair, as the tournament only favors those top two, and not any other skill level. The main idea of the research done for this paper was to determine whether horseshoe tournaments could benefit from the implementation of a seeding and/or reseeding system.

Horseshoes Tournaments

Horseshoe tournaments consist of partners for teams, which has led people to create various ways to select partners and decide which teams play each other. While all horseshoe tournaments either allow people to select their own partners or, more commonly, randomly pick partners, that is where the similarity between the types of tournaments ends. The types of horseshoe tournaments that this paper will examine are Double Elimination, Round Robin, and Horseshoe Mania.

Double Elimination

Although single elimination is more common in sports other than horseshoes, it is more common in a horseshoe setting to have double elimination tournaments. As single elimination tends to be more popular among sports than double elimination, double elimination lacks the amount of research that single elimination has. The principle behind the double elimination tournament is simple: once a team loses twice, they are eliminated. The bracket for a double elimination tournament, however, is not as simple. The tournament starts with a single elimination (also known as "knockout") round, and also includes a second "loser's" bracket underneath it. The loser's bracket consists of teams that have lost once, but not twice. In the initial round, teams 1 and 2 play against each other (match 1), and 3 and 4 do the same (match 2). The winners of each match play against each other, and the losers of the first round do the same. This is further illustrated in figure 1.

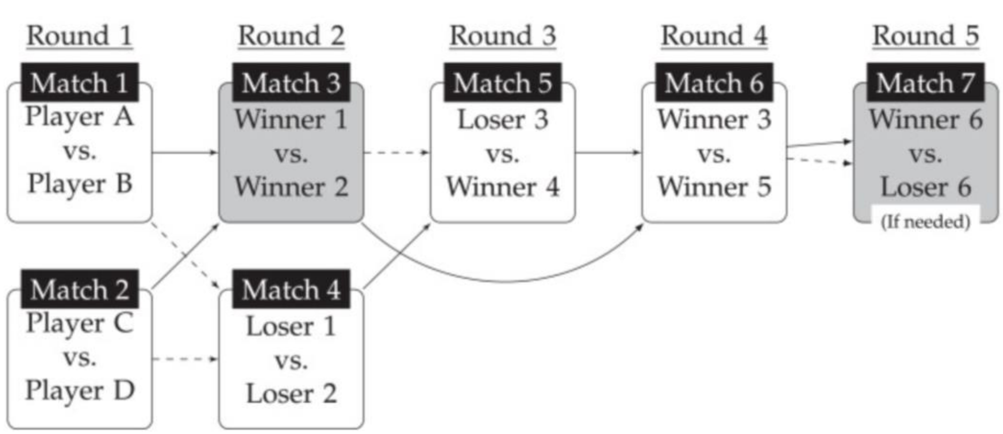


Figure 1: An alternate way of displaying a double elimination bracket with 4 players. Deck, C., & Kimbrough, E. O. (2015). Single- and double-elimination all-pay tournaments. *Journal of Economic Behavior & Organization*, 116, 416–429. https://doi.org/10.1016/j.jebo.2015.05.019

While simple at first glance, this bracket becomes more complicated if the number of teams is not a power of 2. In this case, byes, or matches where teams must wait to play the next round (similar to the waiting gap between matches 3 and 6 in figure 1), are put in place to prevent awkward bracket formations.

The Double Elimination tournament, as it is mainly based on team numbers and not team skill, does not seem very fair. The bracket above seems to favor a seeded tournament, in which the lower numbers have the highest skill, which will be further discussed later in this paper. However, the teams are either randomly drawn or the participants chose their own teams, so this style does not favor skill.

Round Robin

The Round Robin tournament is another common type of tournament. In this version of a tournament, teams are decided at the beginning and all n teams play against each other over $n-1$ rounds. Round Robin is commonly found in FIFA Tournaments or the NFL regular season. The team with the greatest number of wins at the end wins the tournament (Baumann et al., 2010). While this tournament's setup is far easier to understand than double elimination, it is still flawed. For example, if there is a dominant team in Round Robin, the other teams may not put as much effort in as they know they will lose to this team (Krumer et al., 2017, p.1180). This does not reward the skill of the other players; thus, the tournament style is not fair.

Horseshoe Mania

Horseshoe Mania is a type of tournament that consists of a set of 7 games. Each game, participants draw new partners dso that it is highly unlikely the same people match up multiple times. At the end of every game, the total amount of points each team scored is added to both players' points, and whoever has the most points at the end wins (*Greater Victoria Invited to Pitch at Horseshoe Mania*, 2019). Not much is formally researched about the structure of Horseshoe Mania, so its fairness cannot be objectively assessed here. However, since teams are determined by random draw like double elimination, this style of tournament seems to be unfair.

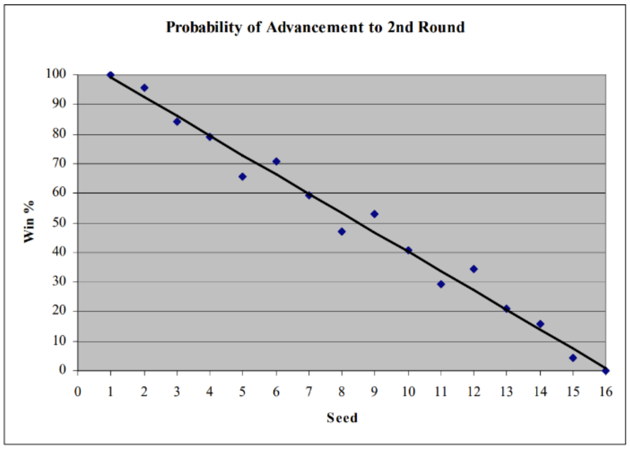
Seeding Tournaments, Reseeding Tournaments, and The Flaws of These Methods

Seeding a tournament helps make the initial round fairer by making a more balanced set of teams and matchups at the beginning. Seeding a tournament is defined as the process of ranking a team by their skill level and designing the structure of the tournament based on this (Schwenk, 2000). To simplify the explanation for seeding, this section will discuss single elimination tournaments to seed, as not much research exists to seed the types of tournaments mentioned in the previous sections. The standard method of seeding a single elimination tournament is set up in a way that is most effective if the number of teams (n) is a power of 2 (let $n = 2^k$). The matches are set up such that the sums of the team numbers in each game for the first round are constant. The following equation shows this setup, where i represents the number of any seed, and $opponent_i$ represent team i's opponent's team number:

$$i + opponent_i = n + 1$$

The standard method favors the teams with higher skill by saving the more interesting matches, where the players of the most skill compete, until the end (Schwenk, 2000).

Despite its benefits in the early part of the tournament, the fairness does not necessarily hold true for the rest of the tournament. While higher skilled players have the highest chance of advancing to the second round, advancing to the third round throws off this balance. As seen in figure 2, a higher amount of skill may not necessarily result in a higher chance of winning for the 3rd round, which is expected and seen in advancing to the 2nd round.



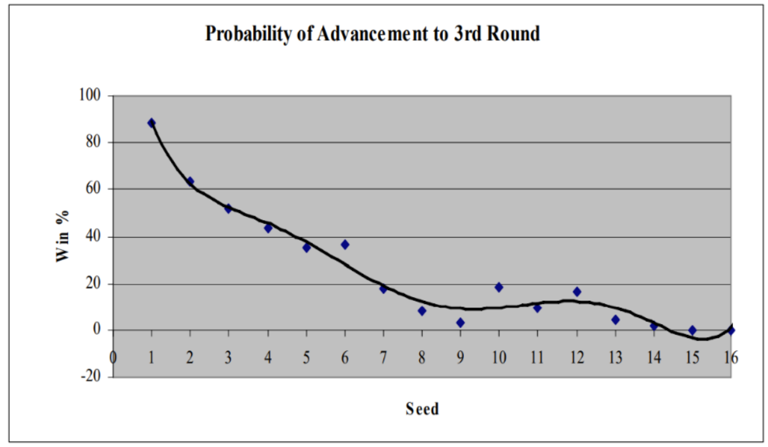


Figure 2: Graphs of the probability of Win % (which is defined as the number of wins / the number of games, taken from different seeded tournaments) vs. Seed. Baumann, R., Matheson, V. A., & Howe, C. A. (2010). Anomalies in Tournament Design: The Madness of March Madness. *Journal of Quantitative Analysis in Sports*, 6(2). https://doi.org/10.2202/1559-0410.1233

The win rate of advancement illustrates the flaw in simply seeding a tournament using the standard method. For example, in advancing to the 3rd round, seed 9 has a far lower chance of advancing to the third round than seed 12. This seed is not fair because team 12 was determined to be less skilled than team 9 from the initial seeding (Baumann et al., 2010).

To resolve this issue, as one article deduces, a tournament organizer can use Cohort Randomized Seeding. The author, similarly to the last article, analyzed this method of seeding with a team number that is a power of 2, however Cohort Randomized Seeding is much more dependent on this than the standard method (Schwenk, 2000). Schwenk (2000) derived this method similarly to how others, which will be detailed later, have derived it: using axioms to show that his method satisfies all of them. These axioms include Delayed Confrontation (the top teams do not play until later rounds), Sincerity Rewarded (no team with higher skill should have a more difficult schedule than a lower seed), and Favoritism Minimized (higher teams should have higher chances of winning) (Schwenk, 2000). This method of seeding separates the teams into *k* Cohorts, with the first two having size $S_1 = S_2 = 2^1$, the third having size $S_3 = 2^2$, the fourth having $S_4 = 2^3$, and so on. The *k*th Cohort will have size $S_k = 2^{k-1}$. The seeds are placed into the cohorts sequentially so that Cohort I has seeds 1 and 2, while Cohort *k* has the following seeds:

$$C_k: \{2^{k-1} + 1, 2^{k-1} + 2, 2^{k-1} + 3, \dots, 2^k - 1, 2^k\}$$

Slots in the bracket are assigned to each cohort, and each team in the cohort is randomly assigned to each slot that corresponds with their cohort (Schwenk, 2000). This bracket is illustrated in figure 3 below.

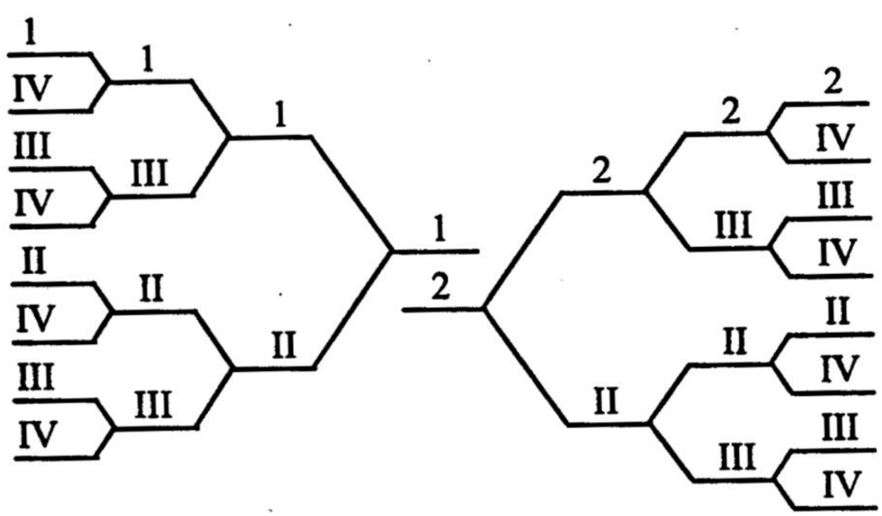


Figure 3: An example of Cohort Randomized Seeding, with 16 teams and 4 Cohorts since $2^4 = 16$. Cohort I consists of teams 1 and 2, but the team numbers are listed for simplicity. Schwenk, A. J. (2000). What Is the Correct Way to Seed a Knockout Tournament? *The American Mathematical Monthly*, 107(2), 140–150. https://doi.org/10.1080/00029890.2000.12005171

Schwenk (2000) proves in his paper that Cohort Randomized Seeding is fairer than the Standard Method by comparing the fairness of the Standard Method to Cohort Randomized Seeding using his axioms. However, this style of seeding also has its flaws. It only seems to be effective for tournaments with a team count that is a power of 2, and it does not consider the expected effort of the players (Schwenk, 2000). Another article analyzes these expected efforts specifically for 4 team tournaments (due to the many various seeding combinations for tournaments that have greater than 4 teams). This article analyzes the 3 different tournament arrangements for 4 teams and tries to maximize effort. Interestingly, the paper's conclusion is that a round like the standard method produces the highest chance of skill being rewarded. They do not mention any extensions about perhaps increasing the number of teams, however this could possibly be used to reseed the semifinals of a tournament (Groh et al., 2008).

Another article claims equal gap seeding is the fairest method. This method of seeding supplies an equal gap between each seed of $G = 2^{k-1}$. In this method, the seeds must follow this equation, where i represents the number of any seed:

$$|i - opponent_i| = G$$

The odd numbered seeds are on one side of the bracket, and the even numbered seeds are on the other. Like Schwenk(2000), the author sets up axioms to prove his/her method is the fairest. These axioms are mostly set up to appeal to spectator interest, such as Delayed Confrontation (explained earlier with Schwenk's article), Increasing Competitive Intensity (matches become harder as the tournament progresses), and Equal Rank Difference (the difference of the seed numbers respectively is constant in the first round) (Karpov, 2016). Comparing the axioms and methods from Schwenk's article to Karpov's article, there is a lot of overlap, but there are differences. For example, the equal gap method seems to follow Delayed Confrontation, but does not follow Favoritism Minimized or the Sincerity Rewarded axiom since in the 16-team example, team 6 must face team 14 but then face team 2 most of the time, whereas team 4 must face 12 but then they will likely face team 8. However, all Karpov's axioms are met by Schwenk's Cohort Randomized Seeding except for Equal Rank Differences.

To provide an alternate method of creating fairness, reseeding reevaluates matchups as the rounds go on. Some sources analyze single elimination tournaments as binary trees to accomplish reseeding. A binary tree is a set of dots, or leaves, branching out from an original dot (the root), which in the case of a single elimination tournament is analyzed from the outermost leaves to the root. Since the opponent of any given team depends on the leaf the team is in, these papers' solutions are a form of reseeding. For example, one article creates a reseeding method where the opponent's team number directly depends on the round number, and the original team's number. In round *r*, with a team count of $n = 2^k$, the opponent of team i, given their opponent had not been previously eliminated, is given by:

$$(opponent)_{\mathrm{i}} = 2^{k+1-r} - i + 1.$$

If the team number is not a power of 2, using this method the organizer could add dummy teams that lose to every real team they face to create this power of 2 (Hwang, 1982). Another article further analyzes reseeding a tournament via binary trees, looking at "antlers" in the trees. Antlers are a type of binary tree that has 2 or more nodes in a bye, and the nodes connect such that there are 6 nodes that are 1 node away from the root, 4 nodes that are 3 nodes away from the root, and 2 nodes that are 2 nodes away from the root. The authors proved that this kind of single elimination cannot exist to favor stronger players (satisfying Monotonicity in Strength). As a corollary of proving this, it is proved that no seeding setup can guarantee stronger players win for the team number is greater than 8 and is a power of 2 (Arlegi & Dimitrov, 2020). It may seem that this contradicts the previous articles, which claimed to have found the best way to make tournaments fairer. However, this article only implies that upsets can always happen for any tournament, which makes it impossible to guarantee that the higher seed always defeats a lower seed.

Conclusion

Although seeding and reseeding tournaments may make the tournament fairer, there are still flaws to be improved upon. The ideal method of seeding single elimination tournaments is debated, with multiple papers each claiming their method to be the fairest. These authors used axiomatic approaches to prove their method was correct, however these axioms varied from paper to paper. A more general definition and axioms for fairness are needed to establish the ideal seeding method.

Further areas of research include seeding Double Elimination tournaments and Horseshoe Mania tournaments, as these are popular tournaments among horseshoe communities. These tournaments may become fairer, and thus benefit from seeding and reseeding. These benefits could be visualized by creating a computer program that seeds the tournaments according to the seeding and reseeding methods in this literature review. The core benefits that each single elimination seeding method provides could be extracted and combined into this computer program to help create even fairer horseshoe tournaments.

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