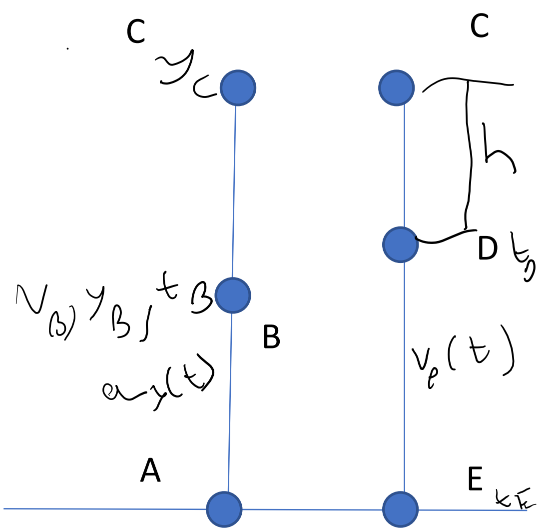
Ian Grzembski October 12, 2020 Section P

Über Rocket Write-Up

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for t_E = 4.5 seconds while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls h = 111m from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.



Givens:

$$t_B = 4.5$$

$$a_y(t) = -1.2t^2 + 20$$

$$v_p(t) = -18\left(1 - e^{\frac{-t}{7}}\right)$$

$$h = 111m$$

Solving the problem:

To find the time it takes for the entire trip, I first found the time for each stage (AB, BD, DE) separately then added these times together. The time for stage AB is given, (4.5 seconds), but the velocity and position at point B are required for stage BD.

Stage AB

To find velocity, I integrate the acceleration for Stage AB:

$$\Delta v_B = \int (a_y)dt$$

$$v_B - 0 = \int (-1.2t^2 + 20)dt$$

$$\underline{v_B(t) = -0.4t^3 + 20t}$$

To find the position at point B, I integrate the velocity for stage AB:

$$\Delta y_B = \int (v_B(t))dt$$

$$y_B - 0 = \int (-0.4t^3 + 20t)dt$$

$$\underline{y_B(t) = -0.1t^4 + 10t^2}$$

Substituting 4.5 for t in both equations, we find the velocity and position:

$$\underline{v_B(t) = -0.4t^3 + 20t}$$

$$v_B(4.5) = -0.4(4.5)^3 + 20(4.5)$$

$$\underline{v_B = 53.55\ m/s}$$

$$\underline{y_B(t) = -0.1t^4 + 10t^2}$$

$$y_B(4.5) = -0.1(4.5)^4 + 10(4.5)^2$$

$$\underline{y_B = 161.494\ m}$$

Stage BD

To find the time it takes to go between points B and D, it is first necessary to know the position of point D. We know that point D is 111m below the max height, so finding the position of point C (which is defined to be the max height) is required. Note that t in this stage is the time since the engines burned out.

$$y(t) = \frac{1}{2}at^2 + v_B t + y_B$$

$$y(t) = \frac{1}{2}(-9.8)t^2 + 53.55t + 161.494$$

$$y(t) = -4.9t^2 + 53.55t + 161.494$$

Note: the maximum of any quadratic equation occurs at the vertex of the parabola it represents. Thus, C is the vertex of the parabola, and we find the maximum height, or y_C.

$$t_C = \frac{-B}{2A}$$

$$t_C = \frac{-53.55}{2 * (-4.9)}$$

$$\underline{t_C = 5.46429\ s}$$

$$y(t) = -4.9t^2 + 53.55t + 161.494$$

$$y(5.464) = -4.9(5.464)^2 + 53.55 * (5.464) + 161.494$$

$$\underline{y_C = 307.80\ m}$$

Knowing that the parachute was given to release 111 meters below the max height, we can write:

$$y_C - y_D = 111$$

$$307.80 - y_D = 111$$

$$\underline{y_D = 196.80\ m}$$

Now that we know the position of point D, we can find the time it took to get there using the original projectile motion equation:

$$y(t) = -4.9t^2 + 53.55t + 161.494$$

$$196.80 = -4.9t^2 + 53.55t + 161.494$$

$$0 = -4.9t^2 + 53.55t - 35.3063$$

Using the N-Spire Solver Function, we get:

$$t_D = 0.704763\ s\ OR\ t_D = 10.2238\ s$$

We only take the second solution because we are only interested in the time when the object is moving downwards.

$$\underline{\boldsymbol{t_D = 10.2238\ s}}$$

Stage DE

To find the time it takes for the object to hit the ground, find when the change in position of the object is down 196.80 meters, because that is the distance to be travelled to the ground. We set the displacement equal to the integral of the given velocity equation:

$$\Delta y_E = \int (v_P(t))dt$$

$$-196.8 = \int_0^t (-18\left(1 - e^{\frac{-t}{7}}\right))\, dt$$

$$-196.8 = -18t - 126e^{\frac{-t}{7}} \Big\{ {t \atop 0}$$

$$-196.8 = -18t - 126e^{\frac{-t}{7}} - (18 * 0 - 126 * e^{\frac{0}{7}})$$

$$-196.8 = -18t - 126e^{\frac{-t}{7}} + 126$$

$$0 = -18t - 126e^{\frac{-t}{7}} + 322.8$$

Using the TI N-Spire's Solver once again, we get:

$$t_E = -9.58178\ s\ OR\ t_E = 17.346\ s$$

We only take the positive root, giving us:

$$\underline{\boldsymbol{t_E = 17.346\ s}}$$

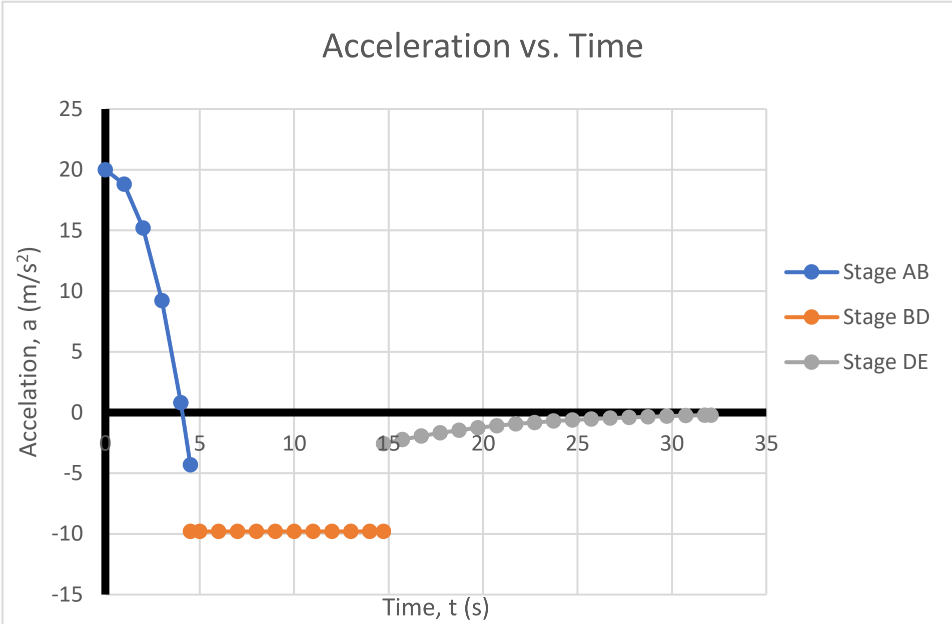
<u>The Final Time</u>

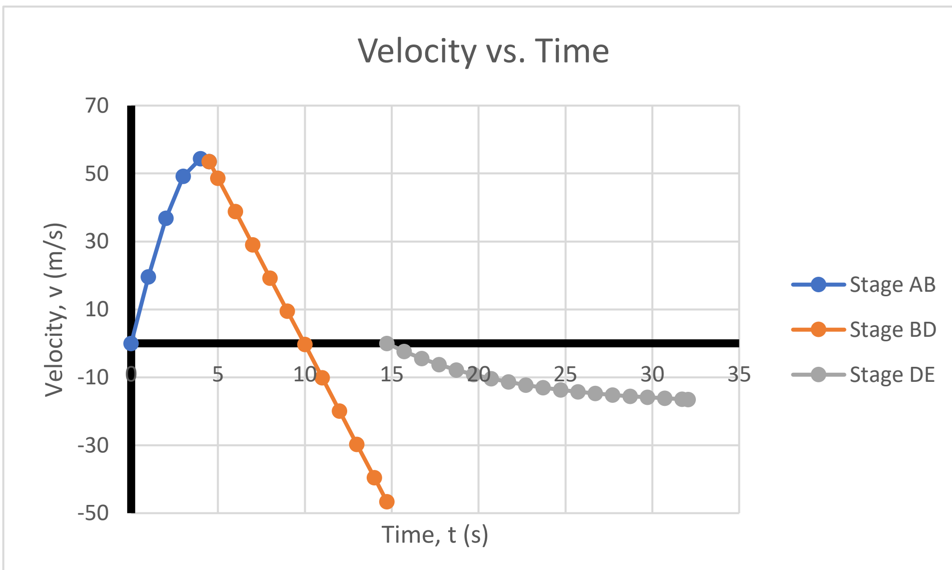
To find the total time the rocket was in flight, add the times of the stages:

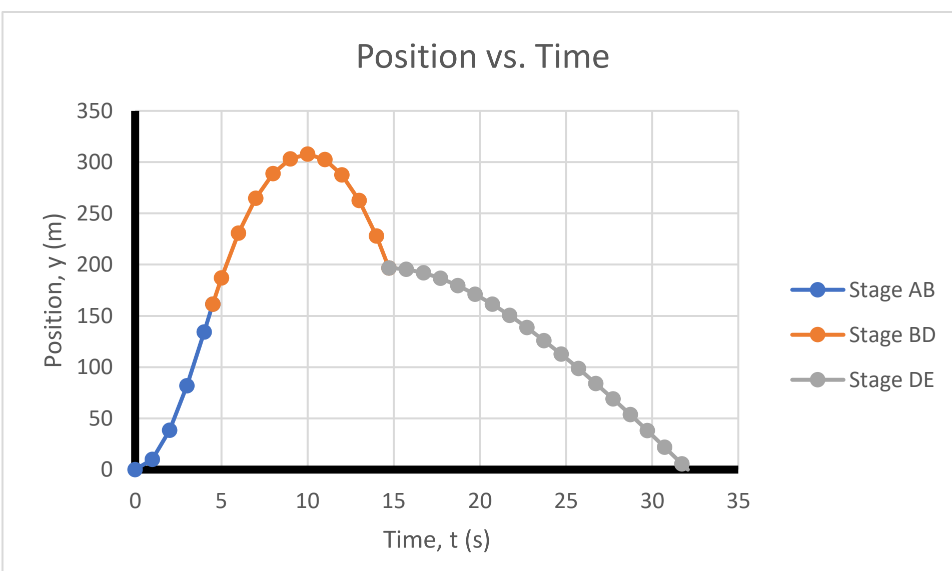
$$t_{tot} = t_B + t_D + t_E$$

$$t_{tot} = 4.5 + 10.2238 + 17.346$$

$$\boxed{\underline{\boldsymbol{t_{tot} = 32.07\ s}}}$$


Acceleration vs. Time
Accelation, a (m/s²)
25
20
15
10
5
0
-5
-10
-15
0
5
10
15
20
25
30
35
Time, t (s)
Stage AB
Stage BD
Stage DE


Velocity vs. Time
Velocity, v (m/s)
70
50
30
10
-10
-30
-50
0
5
10
15
20
25
30
35
Time, t (s)
Stage AB
Stage BD
Stage DE


Position vs. Time
Position, y (m)
350
300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
Time, t (s)
Stage AB
Stage BD
Stage DE