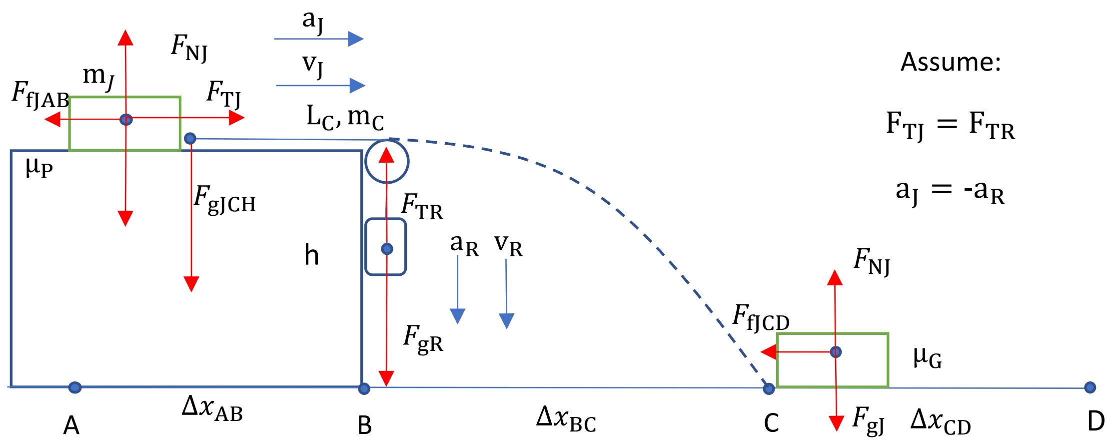
Ian Grzembski November 24, 2020 Section P

"Jerky" Jerry decided to make a jabberwocky jumper of mass $m_J = 68\text{kg}$ using a pulley system (see diagram). His method was to attach one end of a chain of mass $m_C = 41\text{kg}$ and length $L_C = 15\text{m}$ to a barrel of rocks of mass $m_B = 158\text{kg}$, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform of height $h = 25\text{m}$ away from the pulley to point A (the full length of the chain L_C). The platform has a coefficient of friction $\mu_P = 0.16$ with the jumper. When he sat in the jumper, he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his (net) speed into the horizontal direction, and eventually slid to a stop at point D, covering a total horizontal displacement of $\Delta x_{BD} = 101\text{m}$ from the wall of the platform. Find the coefficient of friction μ_G the jumper has with the ground. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.



Stage AB:

The main goal of this stage is to find the velocity that the jumper leaves the platform with (v_B). To do this, the acceleration in terms of position was found based on the mass of the chain in both the horizontal (H) and vertical sections (V) of the system. To find these masses, I set up this proportion, where x is the distance that Jerry has traveled so far on the platform, in meters:

$$\frac{\text{mass of the section}}{\text{length of the section}} = \frac{\text{total mass}}{\text{total length}}$$

$$\frac{m_{CH}}{15\text{-}x} = \frac{41}{15} \quad \text{and} \quad \frac{m_{CV}}{x} = \frac{41}{15}$$

$$\underline{m_{CH} = 41\text{-}41x/15 \quad \text{and} \quad m_{CV} = 41x/15}$$

With these masses, we can solve for the forces of tension for the jumper and barrel. Since these are assumed to be the same, we can set them equal and solve for $a_J[x]$.

Jumper:

$$\Sigma F_{Jy}: F_{NJ} - F_{gJ} = m_J \cdot a_{JyAB}$$

$$F_{NJ} - m_J \cdot g = m_J \cdot 0$$

$$\underline{F_{NJ} = m_J \cdot g}$$

$$F_{fJAB} = \mu_P \cdot F_{NJ}$$

$$\underline{F_{fJAB} = \mu_P \cdot m_J \cdot g}$$

$$\Sigma F_{(J+CH)x}: F_{TJ} - F_{fJAB} = m_{(J+CH)} \cdot a_J$$

$$F_{TJ} - \mu_P \cdot m_J \cdot g = (m_J + m_{CH}) \cdot a_J$$

$$\underline{F_{TJ} = \mu_P \cdot m_J \cdot g + (m_J + m_{CH}) \cdot a_J}$$

Barrel of Rocks:

$$\Sigma F_{Ry}: F_{TR} - F_{g(R+CV)} = m_{(R+CV)} \cdot a_B$$

$$F_{TR} - m_{(R+CV)} \cdot g = m_{(R+CV)} \cdot (\text{-}a_J)$$

$$\underline{F_{TB} = (m_R + m_{CV}) \cdot g - (m_R + m_{CV}) \cdot a_J}$$

Set $F_{TJ} = F_{TB}$.

$$\mu_P \cdot m_J \cdot g + (m_J + m_{CH}) \cdot \boldsymbol{a_J} = (m_R + m_{CV}) \cdot g - (m_R + m_{CV}) \cdot \boldsymbol{a_J}$$

$$(m_J + m_{CH} + m_{CV} + m_R) \cdot \boldsymbol{a_J} = (m_R + m_{CV}) \cdot g - \mu_P \cdot m_J \cdot g$$

$$(m_J + m_C + m_R) \cdot \boldsymbol{a_J} = (m_R + m_{CV}) \cdot g - \mu_P \cdot m_J \cdot g$$

Note that in lines 2 and 3, the sum of the masses of the horizontal and vertical parts is simply the total mass of the chain. I then substitute all the values for the variables, except for x and a_J because those are unknown.

$$(68 + 41 + 158) \cdot \boldsymbol{a_J} = \left(158 + \frac{41}{15}x\right) \cdot 9.8 - 0.16 \cdot 68 \cdot 9.8$$

$$267 \cdot \boldsymbol{a_J} = 1548.4 + 26.7867x - 106.624$$

$$\underline{\boldsymbol{a_J}[x] = 0.100325x + 5.39991}$$

Now that we know the function of acceleration in terms of position, we can use the identity

$$a[x]\,dx = v\,dv$$

which can be derived by multiplying the right side of

$$a \equiv \frac{dv}{dt}$$

by 1, or more specifically $\frac{dx}{dx}$ and rearranging terms. Integrating both sides of the identity, we get:

$$\int_{x_0}^{x} a[x]\, dx = \int_{v_0}^{v} v\, dv$$

$$\int_{0}^{L_C} (0.100325x + 5.39991)\, dx = \int_{0}^{v_B} v\, dv$$

$$\int_{0}^{15} (0.100325x + 5.39991)\, dx = \int_{0}^{v_B} v\, dv$$

I then used the N-Spire to evaluate these two integrals, getting us:

$$92.2852 = \frac{1}{2}{v_B}^2$$

$$\underline{v_B = 13.5857\ m/s}$$

Note we ignore the negative v_B, since Jerry is moving to the right and not to the left.

Stage BC:

The goal of this stage is to find out how far Jerry landed from the platform Δx_{BC}, and his horizontal speed at point C v_C. To find the displacement, we first find out how long it takes for him to land:

$$y_C = \frac{1}{2} a_{yBC} \cdot t_{BC}^2 + v_{By} \cdot t_{BC} + y_B$$

$$0 = \frac{1}{2}(-9.8) \cdot t_{BC}^2 + 0 \cdot t_{BC} + 25$$

$$-25 = -4.9 \cdot t^2{}_{BC}$$

$$\underline{t_{BC} = 2.25877s}$$

We now find the total displacement of the jumper from the platform.

$$x_C = \frac{1}{2} a_{xBC} \cdot t_{BC}^2 + v_{Bx} \cdot t_{BC} + x_B$$

$$\Delta x_{BC} = \frac{1}{2} a_{xBC} \cdot t_{BC}^2 + v_{Bx} \cdot t_{BC}$$

$$\Delta x_{BC} = \frac{1}{2} \cdot 0 \cdot (2.25877)^2 + 13.5857 \cdot 2.25877$$

$$\underline{\Delta x_{BC} = 30.6869m}$$

Next, we find the horizontal speed v_C, which is 75% of his net speed v_{CN}. We can find the x and y components of v_{CN}, then use the Pythagorean theorem to find v_{CN}.

x-dir:

$$v_{Cx} = v_{Bx} + a_{BCx}\Delta t$$

$$v_{Cx} = 13.5857 + 0(2.25877)$$

$$\underline{v_{Cx} = 13.5857\ m/s}$$

y-dir:

$$v_{Cy} = v_{By} + a_{BCy}\Delta t$$

$$v_{Cy} = 0 + (-9.8)(\ 2.25877)$$

$$\underline{v_{Cy} = -22.1359\ m/s}$$

Net Speed:

$$v_{Cx}^2 + v_{Cy}^2 = v_{CN}^2$$

$$(13.5857)^2 + (-22.1359)^2 = v_{CN}^2$$

$$184.57 + 490. = v_{CN}^2$$

$$\underline{v_{CN} = 25.9725\ m/s}$$

Again, we reject the negative value because Jerry is still moving to the right. Multiplying the net speed by 75%, we get that

$$\underline{v_C = \ 19.4794\ m/s}$$

Stage CD:

The main goal of this stage is to find the acceleration of the jumper, which can then be used to find μ_G. To find the acceleration, we first find the velocity, which can be calculated using the total displacement across stage CD:

$$\Delta x_{BC} + \Delta x_{CD} = \Delta x_{BD}$$

$$30.6869 + \Delta x_{CD} = 101$$

$$\underline{\Delta x_{CD} = 70.3131m}$$

$$v_D^2 = v_C^2 + 2a_{CD}\Delta x_{CD}$$

$$0^2 = 19.4794^2 + 2a_{CD}(70.3131)$$

$$0 = 674.57 + 140.626a_{CD}$$

$$\underline{a_{CD} = -2.69826\ m/s^2}$$

Friction:

$$F_{fCD} = \ \mu_G \cdot F_{NJ}$$

$$\underline{F_{fCD} = \ \mu_G \cdot m_J \cdot g}$$

Note we calculated F_{NJ} in stage AB, and it does not change since there are no vertical forces applying in either stage.

Finding μ_G:

$$\Sigma F_{CDx}: -F_{fJ} = m_J \cdot a_{CD}$$

$$-\mu_G \cdot m_J \cdot g = m_J \cdot a_{CD}$$

$$-\mu_G \cdot g = a_{CD}$$

$$-\mu_G \cdot 9.8 = -2.69826$$

$$\boxed{\mu_G = 0.2753}$$