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Math Modeling

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POW #1 Write-up

Problem Statement: We were given a calendar of October and tasked with developing a method of finding the day of the week of any given birthday between 1901 and 2099

Process:

The first thing we looked at was how the days of the week changed each year. The day of the week of a given date moves forward once every year and twice every leap year; therefore, when trying to find the day of the week of a date in the past, the day of the week will shift backward by one every year and two every leap year. Using this information, combined with our knowledge of the number of days in every month, we knew we could find the day of the week of the birthday in the birthday year.

The first step of our process was finding the day of the week of the birthday in the current year. This was done by first finding the number of days between the birthday and the current date, modding by 7 (because there are 7 days in a week) to find the change in day of the week between the two dates, then subtracting from the current day of the week in mod 7 (this way, if the number goes below 0, it will give us the day of the week in the previous week) if the birthday is earlier in the year than the current date, or adding in mod 7 (this way, if the number goes above

6, it will give us the day of the week in the next week) if the birthday comes later than the current date.

The next step of our process was finding the day of the week of the birthday in the appropriate year. To do this, we found the number of years between the current year and the birthday year by subtracting the birthday year from the current year, as well as the number of leap years between that time because that will give us the number of times the day of the week has shifted.

To find the number of leap years between two dates, we started by subtracting the first leap year after the birthday from the last leap year before the current date and dividing it by 4 (because only 1 out of every 4 years is a leap year). Then, we added 1 since the subtraction removes a leap year that should be included in the range. However, to simplify this solution so that the same calculation could be used for dates in the past and in the future and so that the person executing the solution wouldn't have to add 1 at the end, we calculated the number of leap years by subtracting the last leap year before the birthday from the last leap year before the current date. This works because, for dates in both the past and the future, this count includes one extra leap year outside of the range (the leap year before the birthday for dates in the past and the leap year before the current date for dates in the future), eliminating the need to add/subtract one. Furthermore, this calculation results in a value that is positive for past dates and negative for future dates, which creates a unified equation for both past and future dates that subtracts the number of leap years for past dates and adds the number of leap years for future dates.

Once we found the number of years between the current year and the birthday year, as well as the number of leap years, we subtracted that number from the day of the week we found in the first step. That difference gave us the number of times the day of the week had shifted

since the birthday. Modding that number by 7 (because there are 7 days in the week) gave us the index of the day of the week of the birthday (0 for Sunday, 6 for Saturday).

We also considered that if the birthday falls on a leap year, there is a special case. Once February 29 occurs, all the days are shifted forward (February 29 replaces March 1, March 1 replaces March 2,...), so if the birthday occurs before February 29, the birth year should not be used in the calculation of leap year count because the year effectively isn't a leap year yet. However, if the birthday occurs on February 29th, the birthday can be found by doing all the calculations with March 1st and counting the birthday year as a leap year. Then, during the last step, 1 is subtracted from the index (since the resulting index is really March 1), and that will output the day of the week February 29th occurred.

Solution:

Table 1: Order of months with number of days

Number of month	Month name	Days in month
1	January	31
2	February	28 if not a leap year (year cannot be evenly divided by 4) 29 if leap year (year can be evenly divided by 4)
3	March	31
4	April	30
5	May	31
6	June	30
7	July	31
8	August	31
9	September	30
10	October	31
11	November	30
12	December	31

Table 2: Days of the week with indices

Day	Index
Sunday	0
Monday	1
Tuesday	2
Wednesday	3
Thursday	4
Friday	5
Saturday	6

Definitions:

- *Current date: the date it is today.
    - Current month/day of month/year, etc.: the month, day of month, etc. that it is today.
- **Index: The number that refers to the day of the week where 0 represents Sunday and 6 represents Saturday.
    - These numbers can be found in Table 2

Instructions:

Important note: if the birthday is February 29, treat the birthday as if it were March 1 until step 11, then treat it as February 29 from step 11 onward.

1. Using Table 1, determine whether the birthday is earlier or later in the year than the current date*. Months with a greater number come later in the year, and dates with a greater day are later than dates with a lesser day in the same month.
    a. If the birthday is earlier in the year than the current date, continue. Otherwise, skip to step 5.
2. Add the current day of the month to the number of days in each month between the birthday and the current date, including the month of the birthday but not the month of the current date, then subtract the day of the month of the birthday. If the birthday month is the same as the current month, do not count the number of days in that month; only subtract the birthday day of month from the current day of month. The order of the months as well as the number of days in each month can be found in Table 1.
    a. Notation:

$$(Current\ day\ of\ month) + (\sum_{m=birth\ month}^{current\ month-1} days\ in\ month\ m) - (birth\ day\ of\ month)$$

    b. Example: if your birthday was on January 2 and it is October 11, add up 11 + all the days from January to September (you can find those numbers in Table 1) - 2.

3. Subtract this number from the index** of the current day of the week, which can be found in Table 2, then divide the result by 7 and find the remainder. This number is the index of the day of the week of the birthday in the current year.

    a. Notation: $(current\ day\ of\ week\ index\ -\ number\ found\ in\ step\ 2)\ mod\ 7$

4. Skip to step 7.

5. Add the index** of the current day of the week, found in Table 2, to the day of the month of the birthday, and the number of days in each month between the two dates, including the current month but not the month of the birthday, then subtract the current day of the month. If the birthday month is the same as the current month, do not count the number of days in that month; only add the current day of week index and the birthday day of month and subtract the current day of month.

    a. Notation:

$$(current\ day\ of\ week\ index)\ +\ (birthday\ day\ of\ month)\ +\ (\sum_{m=current\ month}^{birth\ month-1} days\ in\ month\ m)\ -\ (current\ day\ of\ month)$$

    b. Example: if you were looking for December 9 and it is Wednesday, September 13, add up 3 (index of Wednesday) + 9 + all the days from September to November (you can find those numbers in Table 1).

6. Divide this result by 7 and find the remainder. This number is the index of the day of the week of the birthday in the current year.

    a. Notation:

    $(number\ found\ in\ step\ 5)\ mod\ 7$

7. If the birth year is equal to the current year, determine the day of the week according to Table 2 using the index of the day of the week of the birthday in the current year (this was calculated in step 3 or 6). This is the final answer. If the birth year is not equal to the current year, proceed.

8. Subtract the birth year from the current year to find the year difference.

    a. Notation: $(current\ year) - (birth\ year) = year\ difference$

9. Determine the year of the most recent leap day:

    a. If the current date is not in a leap year, the year of the most recent leap day is the most recent year that can be evenly divided by 4.

        i. $(current\ year) - ((current\ year)\ mod\ 4) = year\ of\ most\ recent\ leap\ day$

    b. If the current date is in a leap year and the month is either January or February, the year of the most recent leap day can be calculated by subtracting 4 from the current year.

        i. $(current\ year) - 4 = year\ of\ most\ recent\ leap\ day$

    c. If the current date is in a leap year and the month is neither January nor February, the year of the most recent leap day is the current year.

        i. $current\ year = year\ of\ most\ recent\ leap\ day$

10. Determine whether the birthday is in a leap year. If the birth year divides evenly by 4, it is a leap year.

11. Determine the year of the last leap day before the birth date by following the instructions in step 9, but using the year of the birthday rather than the current year.

12. Determine the leap day count using the following formula:

$$\frac{(year\ of\ most\ recent\ leap\ day) - (year\ of\ leap\ day\ before\ birth\ date)}{4}$$

13. If the birthday is in a leap year and is before February 29:

    a. Add one to the leap day count if it's positive

    b. Subtract one from the leap day count if it's negative

14. Determine the index of the day of week of the birthday using the formula:

$$(index\ of\ day\ of\ week\ of\ birth\ date\ in\ the\ current\ year\ -\ year\ difference\ -\ leap\ day\ count)\ mod\ 7$$

    a. mod 7 means to divide the number by 7 and find the remainder. For negative numbers, find the lowest multiple of 7 that can be added to the number to make it positive, then add that to the number.

    b. Example: 9 mod 7 = remainder when 9 is divided by 7 = 2, because 9/7 = 1R2

    c. Example: For -25 mod 7, add the smallest multiple of 7 that makes -25 nonnegative, so -25 + 28 = 3

15. Determine the final answer by mapping this index to a day of the week according to Table 2.

Extensions:

One possible extension of this problem would be a Java program that automates the solution for user-inputted dates

```
import java.time.LocalDate;
import java.util.Scanner;

public class POW1 {
   final static String[] DAYS = {"Sunday", "Monday", "Tuesday", "Wednesday",
"Thursday", "Friday", "Saturday"};
   final static int[] MONTHS = {31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};

   public static void main(String[] args) {
       Scanner sc = new Scanner(System.in);

       LocalDate currentDate = LocalDate.now();
       int currentMonth = currentDate.getMonthValue();
       int currentDay = currentDate.getDayOfMonth();
       int currentYear = currentDate.getYear();
       int currentDayOfWeek = currentDate.getDayOfWeek().getValue();

       System.out.print("Enter your birthday (mm/dd/yyyy): ");
       String birthday = sc.next();

       int birthMonth = Integer.parseInt(birthday.substring(0, 2));
       int birthDay = Integer.parseInt(birthday.substring(3, 5));
       int birthYear = Integer.parseInt(birthday.substring(6));
       if (birthMonth == 2 && birthDay == 29) {
           birthMonth = 3;
           birthDay = 1;
       }
```

```
        int birthDayOfWeekInCurrentYear = 0;
        if (birthMonth < currentMonth || (birthMonth == currentMonth && birthDay <
currentDay)) {
            birthDayOfWeekInCurrentYear = (currentDayOfWeek - daysBetween(birthMonth,
birthDay, currentMonth, currentDay)) % 7;
        } else if (birthMonth > currentMonth || birthDay > currentDay) {
            birthDayOfWeekInCurrentYear = (currentDayOfWeek + daysBetween(currentMonth,
currentDay, birthMonth, birthDay)) % 7;
        }

        int earlierYear = Math.min(birthYear, currentYear);
        int laterYear = Math.max(birthYear, currentYear);

        int yearsBetween = laterYear - earlierYear;
        int leapYearsBetween = 0;
        for (int i = earlierYear; i < laterYear; i++) {
            if (i % 4 == 0) {
                leapYearsBetween++;
            }
        }

        int dayOfWeek = birthYear < currentYear ? (birthDayOfWeekInCurrentYear -
yearsBetween - leapYearsBetween) % 7 : (birthDayOfWeekInCurrentYear + yearsBetween +
leapYearsBetween) % 7;
        if (dayOfWeek < 0) {
            dayOfWeek += 7;
        } else if (birthYear % 4 == 0 && birthMonth < 3) {
            if (birthYear < currentYear) {
                dayOfWeek++;
            } else {
                dayOfWeek--;
            }
```

```
        }


        System.out.println(DAYS[dayOfWeek]);
    }


    public static int daysBetween(int earlierMonth, int earlierDay, int laterMonth, int
laterDay) {
        int daysBetween = laterDay - earlierDay;
        for (int i = earlierMonth - 1; i < laterMonth - 1; i++) {
            daysBetween += MONTHS[i];
        }
        return daysBetween;
    }
}
```

Additional Problems:

- How can you find the weekday of your birthday in the year you were born if you were born before 1901? How can you find the current weekday your birthday will fall on if it is past the year 2099?
- If you were given the year, month, and weekday of your birthday in the year you were born and the current year, can you find a potential list of dates that your birthday could fall on?
- Could you create an app that matches you with people who were born on the same day AND same day of the week as you?