Isaac Lau October 1, 2019 Section J

Uber Rocket – Calculus

Problem description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage. Calculate the total time the rocket is in the air.

Givens:

Acceleration of rocket while engine burns	$a_{AB}[t] = -1.1t^2 + 21$	m/s^2
Engine burn time	$t_{AB} = 4.8$	sec
Vertical distance rocket falls max height before parachute opens	h = 112	m
Velocity of rocket with parachute	$v_{DE} = -18(1 - e^{-t/11})$	m/s
Gravity	$a_{BD} = -9.8$	m/s^2
Initial velocity	$v_A = 0$	m/s
Initial position	$y_A = 0$	m

Variables:

Point A	The rocket is on the ground, right before take off.
Point B	The engine of the rocket cuts out.
Point C	The maximum height of the rocket.
Point D	The point when the parachute is deployed.
Point E	The rocket hits the ground.

Strategy

1. Integrate the acceleration equation into velocity and position equations and substitute the time to solve for velocity (v[t]) and position (y[t]) when the engine cuts out (**point B**).
2. With the velocity and position, use one of the kinematic equations to calculate the maximum height the rocket reaches (**point C**). Subtracting (h = 112m) from this value will give the height when the parachute deploys (**point D**).
3. Using the kinematic equation, calculate at what time the rocket will reach **point D**.
4. Integrate the equation of velocity for when the parachute is deployed (v_{DE}) to find the equation for position (y_{DE}). Take the equation and solve for time when the rocket hits the ground.
5. Add up each subsection for time ($t_{AB} + t_{BD} + t_{DE}$) to find the total time in the air (t_{Total}).

Step 1 (part 1), Solving for v_B:

The equation of acceleration is $a_{AB}[t] = -1.1t^2 + 21$. $v_{AB}[t]$ can be derived by taking the integral of $a_{AB}[t]$.

$$v_{AB}(t) = \int (\ a_{AB})dt$$
$$v_{AB}(t) = \int (\ -1.1t^2\ +\ 21)dt$$
$$v_{AB}(t) = -0.3667t^3 + 21t + C$$

C is 0 m/s because the rocket starts on the ground at rest (v_A = 0 m/s). Substituting 0 for C gives:

$$v_{AB}(t) = -0.3667t^3 + 21t$$

The engine burns for 4.8 seconds, so substituting 4.8 in place of t gives:

$$v_B(4.8) = -0.3667(4.8)^3 + 21(4.8)$$
$$\underline{v_B = 60.2497\ m/s}$$

Step 1 (part 2), Solving for y_B:

The equation of velocity is $v_{AB}[t] = -0.3667t^3 + 21t$. $y_{AB}[t]$ can be derived by taking the integral of $v_{AB}[t]$.

$$y_{AB}(t) = \int (\ v_{AB})dt$$
$$y_{AB}(t) = \int (\ -0.3667t^3\ +\ 21t)dt$$
$$y_{AB}(t) = -0.091667t^4 + 10.5t^2 + C$$

C is 0 m because the rocket starts on the ground (y_A = 0 m). Substituting 0 for C gives:

$$y_B(t) = -0.091667t^4 + 10.5t^2$$

The engine burns for 4.8 seconds, so substituting 4.8 in place of t gives:

$$y_B(4.8) = -0.091667(4.8)^4 + 10.5(4.8)^2$$
$$\underline{y_B = 193.26\ m}$$

Step 2: Find the y_C and y_D

To find maximum height, an equation for y_f needs to be created. The equation:

$$x_f = \frac{1}{2}a\Delta t^2 + v_i\Delta t + x_i$$

Can be modified into:

$$y_c = \frac{1}{2}a_{BD}{t_{BC}}^2 + v_B t_{BC} + y_B$$

Since $a_{BD} = -9.8\ m/s^2$, $v_B = 60.2497\ m/s$, and $y_B = 193.26\ m$, substitute these values into the equation gives:

$$y_C = \frac{1}{2}(-9.8)t_{BC}^2 + (60.2497)t_{BC} + 193.26$$
$$y_C = -4.9t_{BC}^2 + 60.2497t_{BC} + 193.26$$

To find the maximum, $-\frac{b}{2a}$ gives t_{BC} at maximum:

$$t_{BC} = -\frac{b}{2a}$$
$$t_{BC} = -\frac{60.2497}{2(-4.9)}$$
$$\underline{t_{BC} = 6.14793s}$$

Substituting 6.14793 back into the kinematic equation gives:

$$y_C = -4.9(6.15)^2 + 60.2497(6.15) + 193.26$$
$$\underline{y_C = 378.465\ m}$$

The parachute deploys 112 meters from the maximum height, so to find y_D, subtract 112 from y_C:

$$y_D = y_c - 112$$
$$y_D = 378.465 - 112$$
$$\underline{y_D = 266.465\ m}$$

Step 3, Solving for t_{BD}:

The equation to calculate maximum height:

$$y_C = -4.9t_{BC}^2 + 60.2497t_{BC} + 193.26$$

Can be modified to calculate for y_D:

$$y_D = -4.9t_{BD}^2 + 60.2497t_{BD} + 193.26$$

Since $y_D = 266.465$ m, substituting this value into the equation gives:

$$266.465 = -4.9t_{BD}^2 + 60.2497t_{BD} + 193.26$$
$$0 = -4.9t_{BD}^2 + 60.2497t_{BD} - 73.2048$$
$$t_{BD} = 1.367s \text{ or } 10.9289s$$

In this case, 1.367 seconds is not used because that is when the rocket initially reaches 266.465 m. Point D is when the rocket hits 266.465 m for the second time. This gives the time:

$$\underline{t_{BD} = 10.9289s}$$

Step 4, Solving for t_{DE}:

The first part of this step is to create an equation for $y_{DE}[t]$. The equation $v_{DE}[t]$ is:

$$v_{DE} = -18(1 - e^{-\frac{t_{DE}}{11}})$$

The equation of position can be derived by taking the integral of $v_{DE}[t]$:

$$y_{DE}[t_{DE}] = \int (v_{DE})dt_{DE}$$
$$y_{DE}[t_{DE}] = \int (-18(1 - e^{-\frac{t_{DE}}{11}}))dt_{DE}$$
$$y_{DE}[t_{DE}] = \int (-18(1 - e^{-\frac{t_{DE}}{11}}))dt_{DE}$$
$$y_{DE}[t_{DE}] = -18\int (1 - e^{-\frac{t_{DE}}{11}})dt_{DE}$$
$$y_{DE}[t_{DE}] = (-18\int (1)dt_{DE}) - (-18\int (e^{-\frac{t_{DE}}{11}}dt_{DE}))$$
$$y_{DE}[t_{DE}] = -18t_{DE} - 198e^{-\frac{t_{DE}}{11}} + C$$

The next step is to solve for C:

$$y_{DE}[0] = 266.465\ m$$
$$y_{DE}[0] = -18(0) - 198e^{-\frac{(0)}{11}} + C$$

Substitute 266.465 for $y_{DE}[0]$:

$$266.465 = -18(0) - 198e^{-\frac{(0)}{11}} + C$$
$$266.465 = 0 - 198 + C$$
$$C = 464.465\ m$$

Plugging C back into the position equation gives the final equation for $y_{DE}[t_{DE}]$:

$$y_{DE}[t_{DE}] = -18t_{DE} - 198e^{-\frac{t_{DE}}{11}} + 464.465$$

The next step is to find when the rocket hits the ground. Since the ground is at 0 m:

$$y_{DE}[t_{DE}] = 0$$
$$0 = -18t_{DE} - 198e^{-\frac{t_{DE}}{11}} + 464.465$$
$$t_{DE} = -14.2025s \text{ or } 24.6317s$$

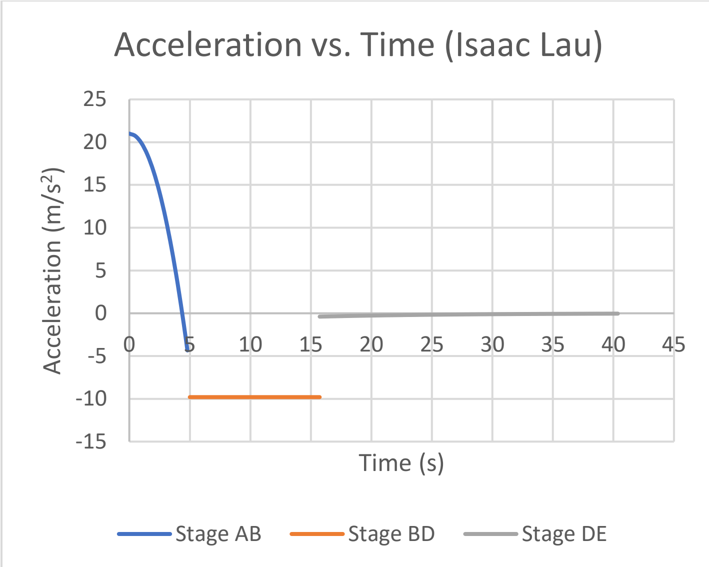
In this case, -14.2025s is not used because the time used is the time after the parachute is deployed.

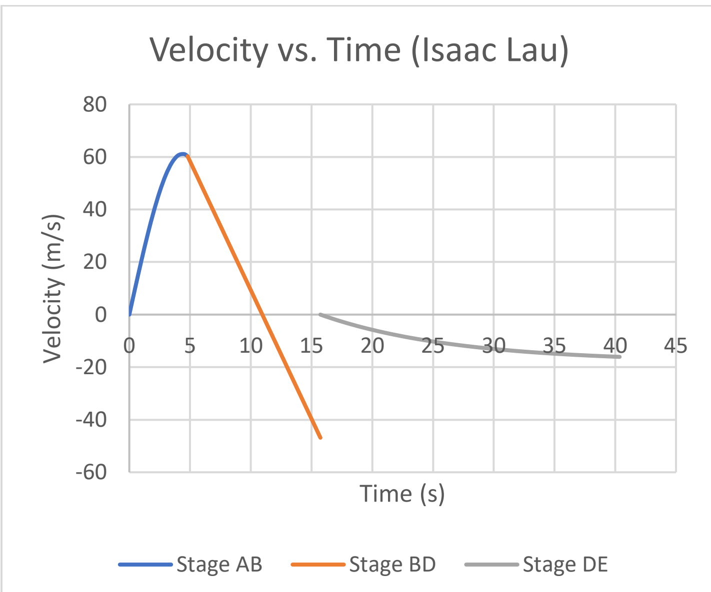
$$\underline{t_{DE} = 24.6317s}$$

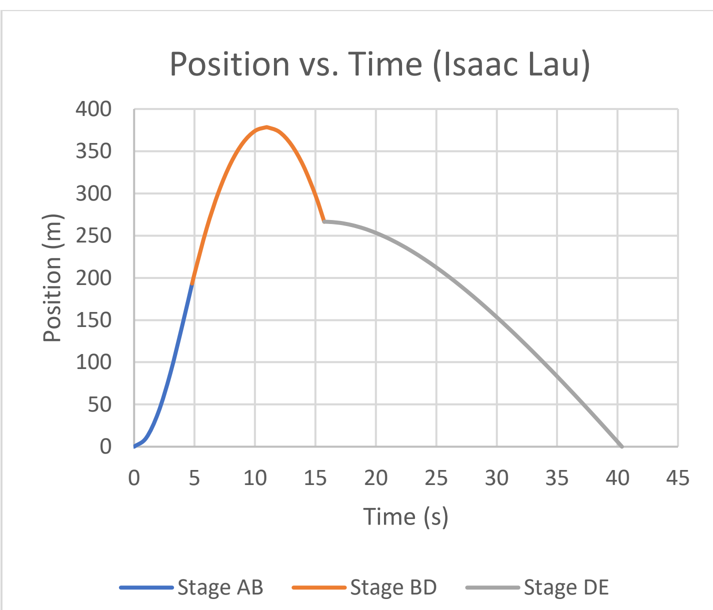
Step 5, find t_T (total time)

For the final step, t_{Total} is found by adding up all of the time segments from before.

$$t_{Total} = t_{AB} + t_{BD} + t_{DE}$$
$$t_{Total} = 4.8 + 10.9289 + 24.6317$$
$$\boxed{t_{Total} = 40.36\ seconds}$$


Acceleration vs. Time (Isaac Lau)
Acceleration (m/s²)
25
20
15
10
5
0
-5
-10
-15
0
5
10
15
20
25
30
35
40
45
Time (s)
Stage AB
Stage BD
Stage DE


Velocity vs. Time (Isaac Lau)
Velocity (m/s)
80
60
40
20
0
-20
-40
-60
0
5
10
15
20
25
30
35
40
45
Time (s)
Stage AB
Stage BD
Stage DE


Position vs. Time (Isaac Lau)
Position (m)
400
350
300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40
45
Time (s)
Stage AB
Stage BD
Stage DE