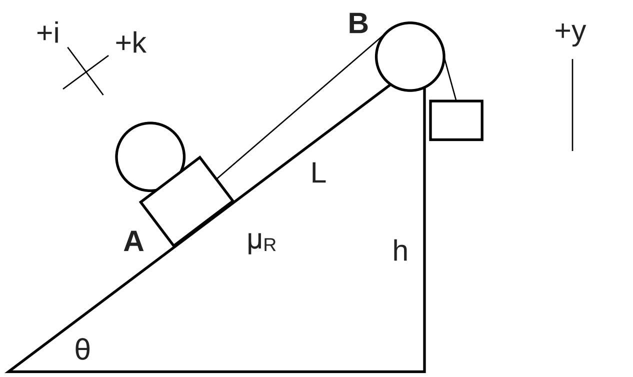
Isaac Lau October 30, 2019 Section J

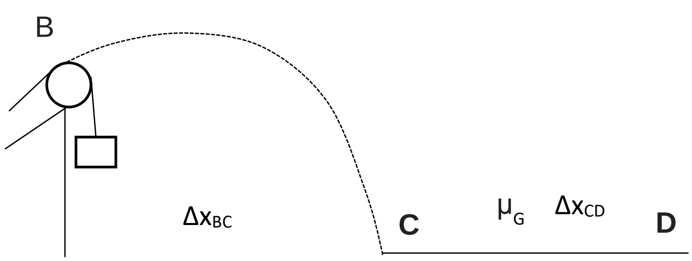
# Uber Pulley – Algebra

**Problem description**

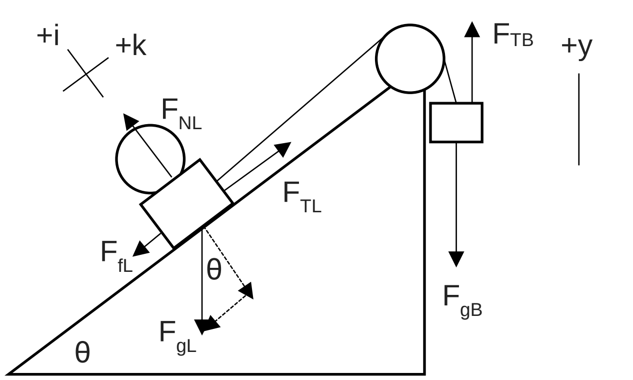
Leaping Larry decided to make a laborious launcher for his luxury luge using a pulley and ramp system. His method was to attach one end of a massless stretchless rope to a barrel or rocks and to hold the other end of the rope. He placed the rope over a massless frictionless pulley, and then walked down the ramp as far down as possible to point A (where L = h). When he sat in the luge he accelerated up the ramp to point B and then launched off the top at the same angle as the ramp (all while releasing the rope and avoiding the pulley). He flew through the air as a projectile to point C, transitioning all of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any height differences between luge height, barrel height, and size of the pulley, and the diagram is not drawn to scale.

**Diagrams:**




**Free Body Diagram:**




**Givens:**

| | |
|---|---|
| Total mass of Larry and luge | $m_L = 40$ kg |
| Total mass of barrel and rocks | $m_B = 55$ kg |
| Ramp angle above the horizontal | $\theta = 34$ deg |
| Coefficient of friction between luge and ramp | $\mu_R = 0.13$ |
| Height of ramp | $h = 6.5$ m |
| Total horizontal distance from vertical base of ramp to final location | $\Delta x_{BD} = 57$ m |
| Coefficient of friction between luge and ground | $\mu_G = ???$ |

**Variables:**

| | |
|---|---|
| Point **A** | When Larry and the luge is at the lowest point on the ramp |
| Point **B** | The point Larry and the luge reaches the pulley |
| Point **C** | The point where Larry and the luge hits the ground |
| Point **D** | The point where Larry and the luge slides to a stop |

**Strategy**

1. Find the acceleration of Larry and the luge from the equations from the force of friction, force of gravity, normal force, and force of tension from the rope at **point A**.
2. With the acceleration of Larry and the luge, calculate their velocity at **point B**.
3. Componentize the velocity into its x and y axis and calculate the total x distance traveled right before Larry and the luge hit the ground at **point C**.
4. Find the net velocity of Larry and the luge at **point C** and find the acceleration of Larry and the luge so that they travel $\Delta x_{CD}$ to **point D**.
5. With the acceleration, find the coefficient of friction ($\mu_G$) from **point C** to **point D** between the luge and the ground.

**Step 1, Solving for $a_A$:**

Some pre-equation assumptions:

$$F_{TL} = F_{TB}$$
$$a_L = -a_B$$

The sum of the forces in the j direction are:

$$\boldsymbol{\Sigma F_k}: F_{TL} - F_{fL} - F_{gL}\sin\theta = \mathrm{m_L} a_L$$

Solving for $F_{T1}$ gets:

$$F_{TL} = \mathrm{m_L} a_L + F_{gL}\sin\theta + F_{fL}$$

The sum of the forces in the y axis is:

$$\boldsymbol{\Sigma F_y}: F_{TB} - F_{gB} = m_B a_B$$
$$F_{TB} = m_B a_B + F_{gB}$$

Since that $F_{T1} = F_{T2}$, the two equations can be set equal to each other:

$$\mathrm{m_L} a_L + F_{gL}\sin\theta + F_{fL} = m_B a_B + F_{gB}$$

$F_f$ comes from $F_N$, which is in the i axis:

$$\boldsymbol{\Sigma F_i}: F_{NL} - F_{gL}cos\theta = \mathrm{m_L}a_L$$

There is no acceleration in the i axis because the luge stays on the ramp.

$$F_{NL} - F_{gL}cos\theta = 0$$
$$F_{NL} = F_{gL}cos\theta$$

The equation for force of friction is:

$$F_{fL} = \mu_R F_{NL}$$

Substituting $F_{NL}$ into this equation gives:

$$F_{fL} = \mu_R m_L g cos\theta$$

Substituting all of this into the $F_{T1} = F_{T2}$ equation gets:

$$\mathrm{m_L}a_L + m_L g sin\theta + \mu_R m_L g cos\theta = m_B a_B + m_B g$$

Since $a_1 = -a_2$, change $a_2$ to $-a_1$:

$$\mathrm{m_L}a_L + m_L g sin\theta + \mu_R m_L g cos\theta = -m_B a_L + m_B g$$

Substitute the values and solve for $a_L$:

$$(40)a_L + (40)(9.8)\sin(34) + (0.13)(40)(9.8)\cos(34) = -(55)a_L + (55)(9.8)$$
$$40a_L + 219.204 + 42.25 = -55a_L + 539$$
$$95a_L = 277.549$$
$$\underline{a_L = 2.922\ m/s^2}$$

$a_L$ is the same acceleration for all of when Larry and the luge is on the ramp.

**Step 2, Solving for $v_B$:**

Using the kinematic equation:

$$v_B^2 = v_A^2 + 2a\Delta x$$

Δx on the ramp is the same as the height, as noted in the problem description. The initial velocity is also 0.
Substitute the values into the equation and simplify:

$$v_B^2 = 0^2 + 2(2.92156)(6.5)$$
$$v_B^2 = 37.9803$$
$$\underline{v_B = 6.163\ m/s}$$

This is the net velocity at point B.

**Step 3, Solving for $\Delta x_{BC}$:**

First, solve for the initial vertical and horizontal velocity:

$$v_{xB} = v_B sin\theta$$
$$v_{xB} = 6.163 sin 34$$
$$v_{xB} = 3.446\ m/s$$

$$v_{yB} = v_B cos\theta$$
$$v_{yB} = 6.163 cos 34$$
$$v_{yB} = 5.109\ m/s$$

Find the total time Larry and the luge spend in the air with the kinematic equation:

$$y_f = \frac{1}{2}a_y t^2 + v_{yB}t + y_i$$

Substitute and solve:

$$0 = \frac{1}{2}(-9.8)t^2 + (3.4462)t + 6.5$$
$$0 = -4.9t^2 + (3.4462)t + 6.5$$
$$t = -0.852s \text{ or } 1.556s$$

Only use the positive value in this case so:

$$t = 1.556s$$

The horizontal velocity is constant, so use the kinematic equation to solve for total horizontal distance traveled:

$$\Delta x_{BC} = v_x t$$
$$\Delta x_{BC} = (5.109)(1.556)$$
$$\underline{\Delta x_{BC} = 7.949m}$$

**Step 4, Find net $a_{CD}$:**

Larry and the luge transfer all of their net velocity at point C into horizontal velocity. Find vertical velocity:

$$v_{yC} = a_y t + v_{yB}$$
$$v_{yC} = (-9.8)(1.556) + (3.446)$$
$$v_{yC} = -11.80\ m/s$$

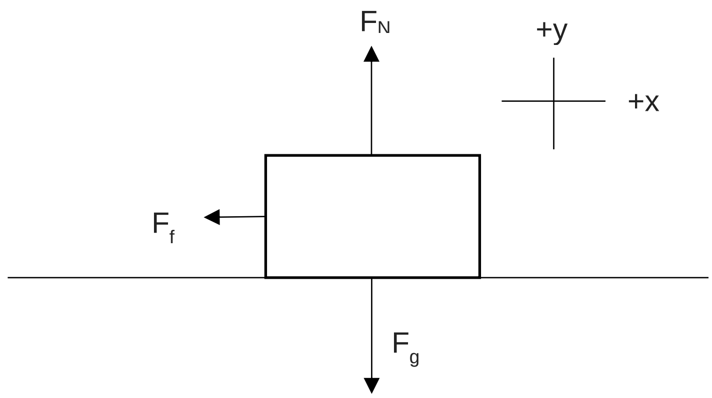
Use the Pythagorean theorem to find the net velocity:

$$v_y^2 + v_x^2 = v^2$$
$$(-11.80)^2 + (5.109)^2 = v^2$$
$$165.38 = v^2$$
$$v = 12.86\ m/s$$

To find the acceleration from point C to D, use the kinematic equation, substitute, and solve for $a_{CD}$:

$$v_D^2 = v_C^2 + 2a_{CD}\Delta x_{CD}$$
$$0^2 = 12.86^2 + 2a_{CD}(57 - 7.949)$$
$$0 = 165.38 + 98.102a_{CD}$$
$$\underline{a_{CD} = -1.686\ m/s^2}$$

**Free Body Diagram for Step 5:**




**Step 5, Solve for $\mu_G$:**

The sum of the forces in the y-axis is:

$$\boldsymbol{\Sigma F_y}: F_N - F_g = ma$$

Larry and the luge remain on the ground, so acceleration is 0.
Solving for $F_N$ gives:

$$F_N = F_g$$
$$F_N = mg$$

The equation for friction is:

$$F_f = \mu_G F_N$$

Substituting in $F_N$ gives:

$$F_f = \mu_G mg$$

The sum of the forces in the x-axis is:

$$\boldsymbol{\Sigma F_x}: -F_f = ma$$

Substituting $F_F$ gives the final x-axis equation:

$$-\mu_G mg = ma$$

Substituting numbers and solving for $\mu_G$ gives:

$$-\mu_G(40)(9.8) = (40)(-1.686)$$
$$-\mu_G(392) = -67.44$$
$$-\mu_G = -0.1720$$

**$\mu_G$ = 0.1720**