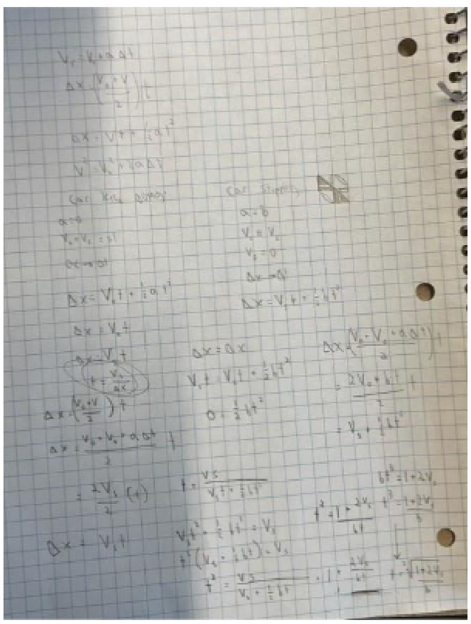
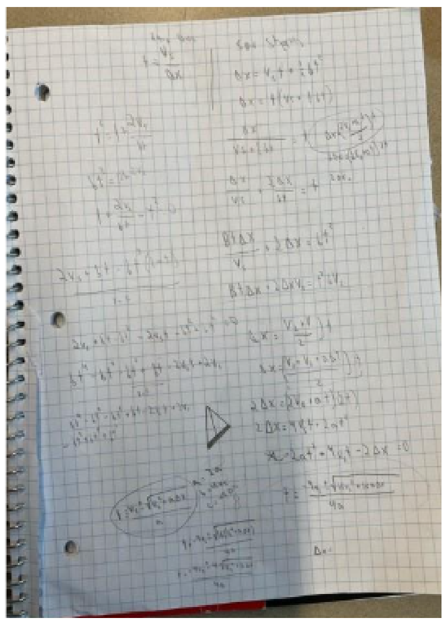
Problem Statement:

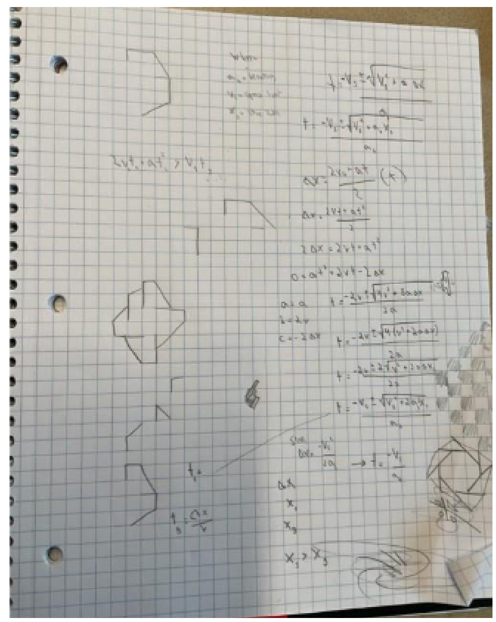
We must develop a formula for the duration of a yellow light at a traffic intersection. In order to come up with a reliable formula, we must factor in many variables that account for the conditions of the intersection. We are given the information below:

- Cars approaching an intersection travel at the speed limit.
- A car proceeding through an intersection must make it entirely through the
- intersection while maintaining the speed limit before the light turns from yellow to red.
- A car whose driver has made the decision to stop must be able to safely stop the car before entering the intersection, i.e., before the stop line of the intersection.

Process:







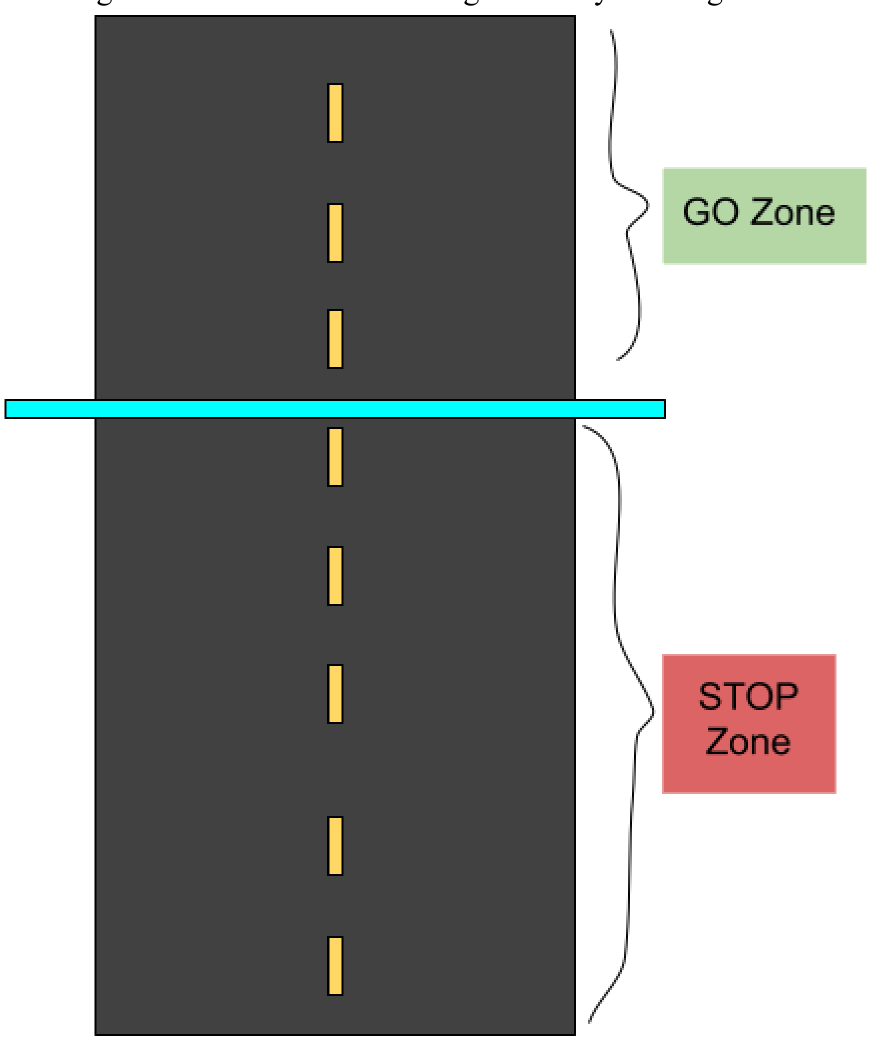
We made two equations for the distance of the STOP zone and the GO zone, where, for going, a=0 (the drivers are passing through the intersection at constant velocity) and $v_0 = v = v_s$ (speed limit velocity), and Δx represented the length of the GO zone. For stopping, a=a_b (braking acceleration) $v_0 = v_s$, $v = 0$. The car-stopping equation became $\Delta x = v_s + \frac{1}{2}a_b t_b^2$, and the car-going equation became $t_g = \frac{\Delta x}{v_s}$, and both of these equations were made from modifying the Big 4 kinematic equations with constant acceleration. We set these equal to each other and got $0 = \frac{1}{2}a_b^2$, which, for obvious reasons, didn't work. We kept trying this, finally revealing t (1+2vs/b)^1/3 at which point we realized we not only did the algebra wrong but the distances are not equal. From these efforts, we realized that the equations for the yellow light time for the

STOP and GO zones could be derived separately rather than jointly, so we continued to use the equation for t_g, which remained robust.

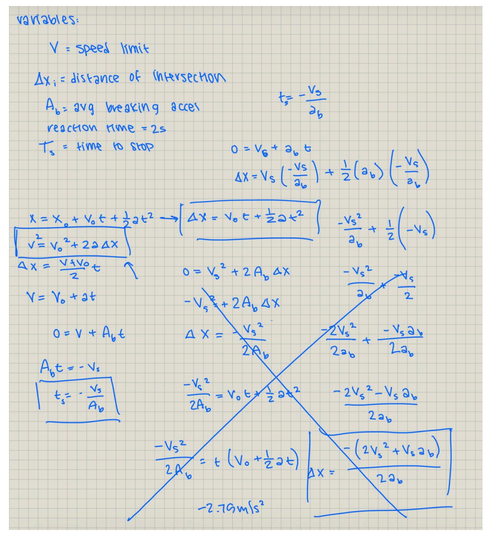
Then, as seen in the work, we transitioned to isolating t for both stop and go situations, where the go zone and stop zone were separate. It took a lot of work to isolate ts (stopping time), but when x is substituted, many terms are cancelled out.

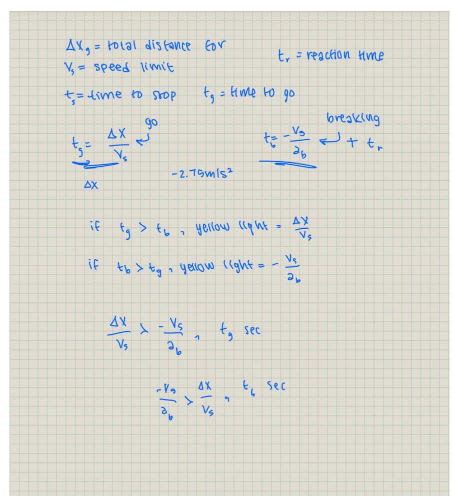
Solution:

1. We noticed that this problem has two scenarios: the car approaching the yellow light is in the go zone and must pass through the light, or the car approaching the yellow light is in the stop zone and must stop before the intersection. The length of the yellow light must account for both, meaning it must provide enough time for both scenarios. This means that if we can find an equation that represents time for each scenario, the one with the greatest time must be the length of the yellow light.



2. We started off our last calculations by recognizing that the car that goes through the yellow light moves at a constant velocity. Using this information, we constructed the formula below:
   a. $t_g = \Delta x / v_s$
      i. t_g = time it takes for the car to go through the intersection
      ii. Δx = length of intersection + go zone
      iii. v_s = speed limit (velocity at which the car travels)
      iv. The above formula is derived from the formula for constant velocity ($\Delta x = v_0 t + \frac{1}{2}at^2,\ a = 0$): $\Delta x = v_s t$. We divided Δx by v to isolate t, thus creating a formula for the time the car needs in the go zone.
      v. (Figure 1)
3. Then, we were left with the task of finding the equation that represents the car stopping. We utilized the format of the following known kinematic equation: $v = v_o + at$. Using our variables relating to the POW, the equation can be rewritten as:
   a. $0 = v_s + a_b t_b$
      i. 0 = v because the final velocity of the car in the stop zone should be 0, at a complete stop.
      ii. Vs in our equation represents initial velocity, the speed limit
      iii. a_b= average acceleration of a braking car
      iv. t_b= time it takes for the car traveling at the speed limit to come to a complete stop.
   b. From here, we manipulated the equation to isolate t_b.
      i. Subtracted v_s from both sides to get $-v_s = a_b t$.
      ii. Divide by a_b to get $t_b = \frac{v_s}{2a_b}$
      iii. (Figure 1)
4. These two equations are applicable to any intersection. Dimensions of the intersection can be inserted into the equations. Whichever equation results in a greater time, the yellow light will follow that number. (Figure 2)
5.





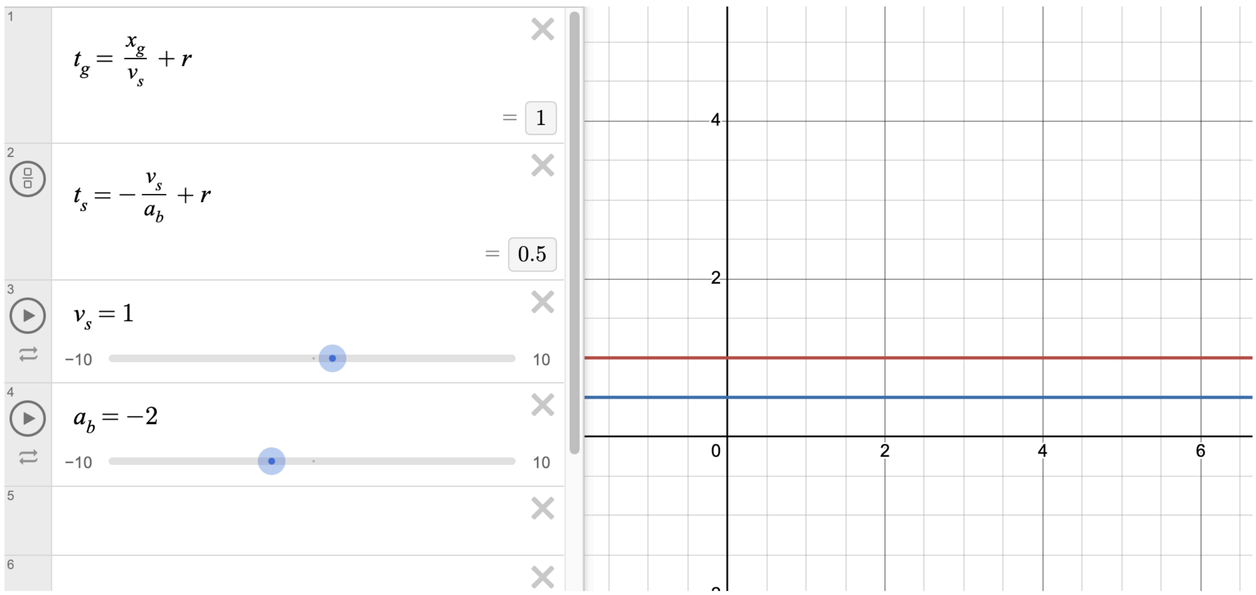
Thus, given $t_g = x_g/v_s$ and $t_s = -v_s/a_b$,

If $t_g > t_s$: $t = x_g/v_s + r$

If $t_s > t_g$: $t = -v_s/a_b + r$

Where:

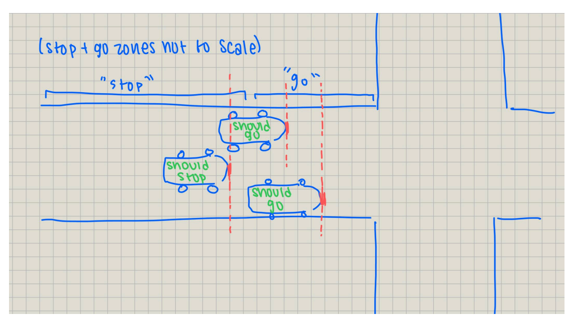
- t_g is go time
- t_s is stop time
- x_g is the distance of the car
- v_s is the speed limit speed
- a_b is the breaking acceleration
- r is reaction time



**In the above Desmos representation, you choose the higher line, which represents one of two possible stopping times*

Discussion Questions:

A. The STOP and GO zones must follow the following rules to be safe: The STOP zone ends where the GO zone begins because the cars are travelling at a constant acceleration, so there can not be an overlap between the STOP and GO zones. We decided that the front of the car determines if the car is considered in a certain section, as demonstrated in the image to the right:



B.

a. Exceeding the speed limit would affect the STOP and GO zones as v_s would become larger, so the time for going through the intersection would increase as per the equation $t_g = \frac{\Delta x}{v_s}$. The time for going would decrease, meaning that the length of the GO zone could also decrease because the same distance is now being covered with more velocity. If a driver is exceeding the speed limit, then the length of the STOP zone would have to increase, as the time spent decelerating would need to increase to bring a greater speed to zero. As per the equation $t_b = \frac{v_s}{2a_b} + t_r$, where t_b represents the time needed to successfully brake before a red light, and a_b represents the acceleration of a car while braking, an increase in the speed limit (v_s) would result in an increase in t_b. Overall, the yellow light time would increase when the speed limit increases because people in the STOP zone require more time to come to a full stop.

b. Bald tires would affect the yellow light by requiring a longer yellow light for drivers at the STOP zone. In the equation$t_b = \frac{v_s}{2a_b} + t_r$, the acceleration for braking (a_b) would decrease, meaning that t_b becomes larger.

c. A long vehicle, such as a semi-truck, would increase the required yellow light time because Δx (distance of the GO zone) would increase with a larger vehicle, as the vehicle has to cover a longer distance than a smaller car to cross the same intersection. In the equation $t_g = \frac{\Delta x}{v_s}$, when Δx is greater, t_g would also increase.

d. A distracted driver would increase the required time of the yellow light because the reaction time (t_r) would increase. In the equation $t_b = \frac{v_s}{2a_b} + t_r$, t_r, the reaction time would increase, resulting in an increase of t_b, the time required for braking. The length of the zones would not be affected as a result of an increase in t_r because reaction time is not a factor in the equations for the distance of either zone.

C. It would be a good idea to mark the stop and go zones at the intersection. This will reinforce to the driver the timing of the yellow light, and it allows us to trust the algorithm more without worrying that a driver will behave unexpectedly in their situation.

D. This idea was rejected because it encourages drivers to race against the clock, the time displayed providing a false sense of confidence. Accidents become more prevalent with reckless driving and high accelerations across intersections; a number of drivers and civilians most likely suffered from this. Moreover, if the times were to change based on a new circumstance (like a speeding car entering the intersection) the other cars would not be ready and unprepared for extra waiting time.