Traffic Light POW

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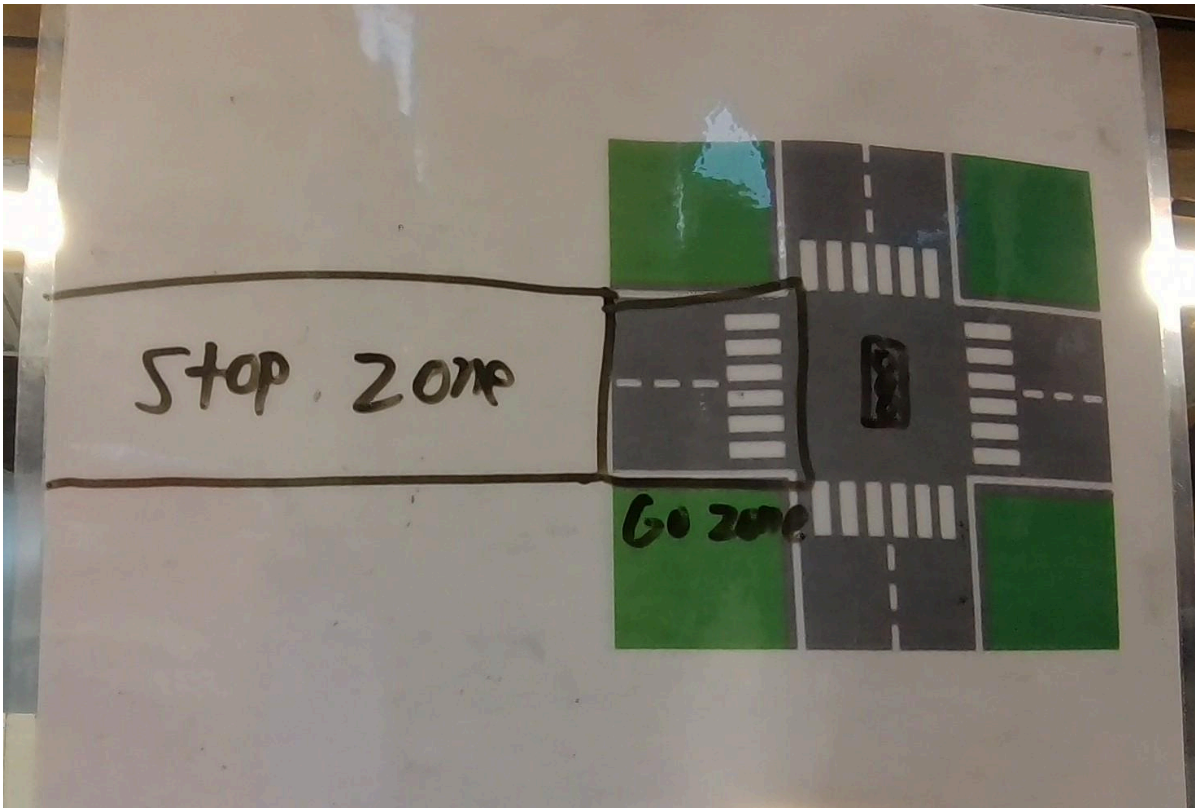
Problem Statement

In our problem, we were tasked with creating a formula to calculate how long a traffic light should stay yellow and what factors affected this decision. The assumptions we were given were: all cars approached the intersection at the speed limit, a car continuing to drive during the yellow light must be able to proceed through the entire intersection while maintaining the speed limit before the light turns from yellow to red, and a car that is stopping must be able to stop before entering the intersection. Our problem contains some important terminology. The "Go Zone" highlights the region in which it is safe for a car to continue driving through the light and make it to the other side before the light turns red. The "STOP Zone" is the region where a car begins to brake at a yellow light and is able to come to a stop without entering the intersection before or when the light turns red.



Process

Before beginning to work on the problem we decided to create a list of variables and decide which of them are constants and which of them we need to be able to calculate or find through a formula. The variables we came up initially are as follows:

- Speed Limit
  - Constant.
- Time of Yellow Light
  - What we are trying to find,
    - Unknown.
- Distance of the Intersection

- Constant.
- "Go Zone" Size
  - Constant.
  - Unknown.
- "STOP Zone" Size
  - Constant.
  - Unknown.
- Acceleration
  - Size of the vehicle factors into.
- Reaction Time
  - Constant

Looking at the problem, we came up with parameters for some of the variables. Since we are working on the assumption that all cars approaching the intersection are going the speed limit, which means their velocity is always at the speed limit, we realized acceleration in that case is zero. When a car is braking, however, it means that the car is decelerating, or in other words, its acceleration is negative and velocity is approaching 0. To simplify the problem, we decided to take size of the vehicle out of the equation, as otherwise, we would need to account for size of vehicle when working on acceleration, but size of vehicle changes per car, meaning the yellow light time would have to change for every single vehicle. This is impossible and convoluted, so we either had the option to take an average vehicle weight or neglect it all together, and we decided to go with the latter.

This means that acceleration is a constant for every single vehicle, either being negative as it is decelerating and approaching the intersection, or zero as it is traveling at speed limit towards and through the intersection. This has simple implications for velocity, which is that velocity will either always be the speed limit as it approaches and moves through the intersection, or approaching zero as it decelerates. Also, we decided to further simplify the problem by saying a car at rest will immediately start travelling at the speed limit. This means we never have to think about positive acceleration. And further, every car will start moving and braking at the same time.

During the beginning of our brainstorming we were very confused about what the STOP zone and GO zone were, but after thinking it through and getting some assistance from Mrs. Chase, we concluded: the STOP zone should be everything before a certain line, where, after passing the line, the car cannot stop before the intersection. Ideas of the STOP zone having an outerbound did come up, but this was put down as a car at any point on the road that comes to a halt before the light turns red and entering the intersection is technically within the STOP zone, even if it's a mile down the road. To know when the car should start braking before being at the last possible place before being within the intersection when decelerating, we need to know deceleration itself. The GO zone is going to be dependent on the time of the yellow light, as the longer the yellow light, logically, the more cars that should be able to make it through the intersection before the light turns red. The GO zone and STOP zone are a bit confusing, as we have three theories for it: they will overlap, they will have a gap, or they will be side by side. At this point, we believe they will overlap.

Given all this information, we can now try to calculate how long it takes the car to stop. This is significant as we believed the time in which it takes the car to stop should be the yellow light time, as a car will only need as much time on a yellow light as it takes to stop.

Taking what we know and turning them into variables for an equation:

- v_0 = speed limit/car's velocity as it starts braking.
- v_f = 0, as the car will be at rest when it finishes braking.
- t_0 = reaction time, as the car will start decelerating when the driver reacts.
- t_f = when the car comes to a complete stop. Unknown.

- Δt = time to brake, which logically is just time to stop minus reaction time.

In the picture, we also define x_0 to be the GO zone start, but that isn't too important to the following process.

Now that we have defined our variables, we can use them to find the time to stop. We decided to use the kinematic equation $v = v_0 + at$ as it doesn't have x in it, the variable that we lack. Plugging in values,

$v = v_0 + at$

$0 = v_0 + a(t_f - t_0)$

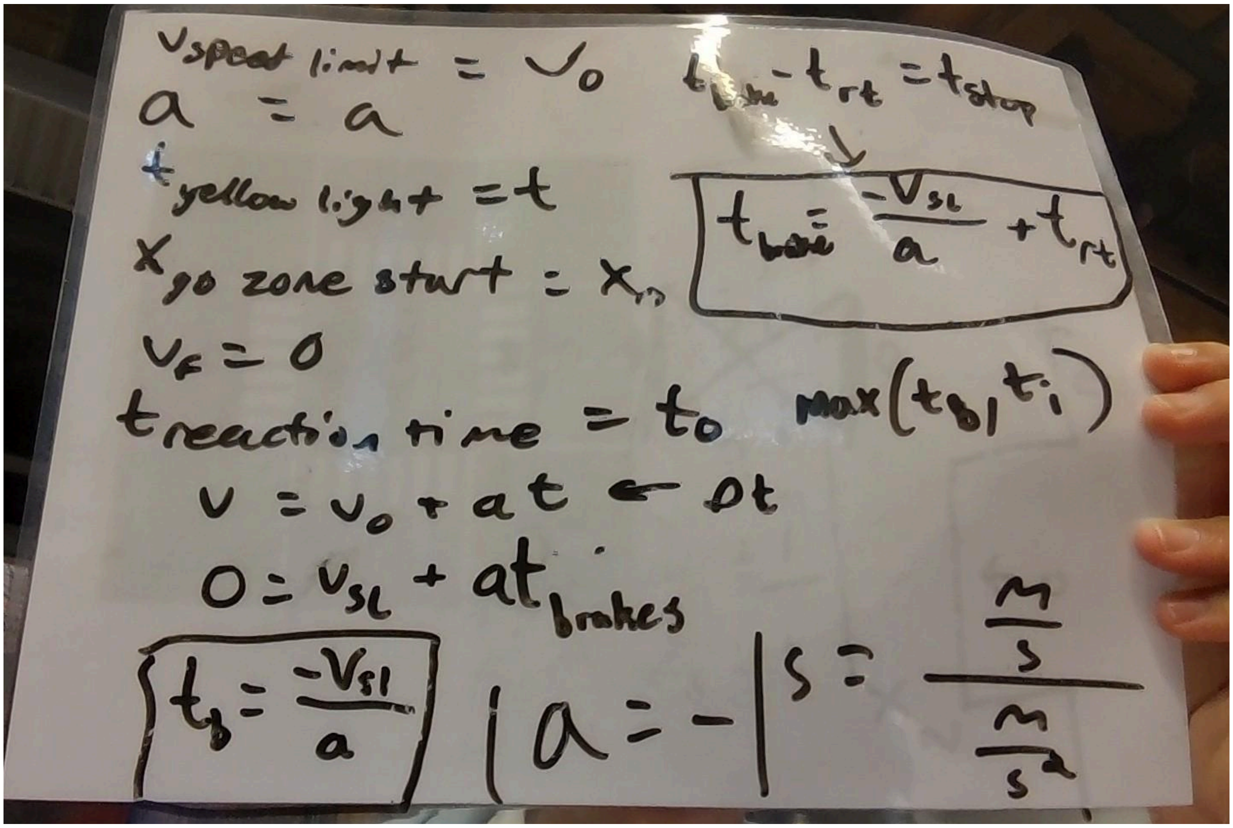
$-v_0 = a(t_f - t_0)$

$-v_0 / a = t_f - t_0$

$-v_0 / a + t_0 = t_f$.

Which in words sounds something like this: the time the car takes to stop is equal to the time it takes the car to decelerate plus the driver's reaction time.

Note, the image below which contains our process is labeled incorrectly. In the top right corner we write $t_{brake} - t_{rt} = t_{stop}$, which is incorrect, being the same as writing with our variables $\Delta t - t_0 = t_f$, and to really drive the point home, $\Delta t = t_f + t_0$. The equation we write below isn't actually wrong, as we cancel out the mistake by subbing out t_{stop} for the equivalency of t_{brake}, giving us the right equation, just mislabeled to be t_{brake} rather than t_{stop}.



We found out how long a car takes to stop, so the traffic light should just be equal to this to give cars ample time to stop, so we’re done, right? Not quite actually. This would mean that we would base yellow light time purely on how long it takes a driver to stop. This may seem fine at first, but not when we are working on the assumption that a car is only in the GO zone if they can make it from where they are to the other side of the intersection before the light turns red. Our equation may end up resulting in a time too short if we don’t actually use this assumption in the equation, thus our new goal is to incorporate into the equation the time it takes the car to cross the intersection. This brings in a new variable, the size of the intersection.

Let’s take a car that is within the STOP zone at the time the light turns yellow. This car, abiding by all of our assumptions and rules, let’s say it stops right before the intersection right as the light turns red. That is, the very last point before the car is deemed to be within the intersection.

This situation is fine, but let's take a situation in which the car was just a bit further up the road than it was just now. Since deceleration is constant and the car was at the last point before the intersection previously, being further up the road it is forced to go further than it had before. Even if the car put on its full brakes, as it is slightly further than before when it ended just outside the intersection on full brakes, this time it will end up in the intersection. But this is a problem, because it means that the car will now be in the intersection just as the light turns red. This means that if the car is decelerating, it won't safely be before the intersection, meaning that it instead must keep going. The only issue with this case is that there may be a gap between the GO zone and the STOP zone in the case that the same car isn't able to make it through the entire intersection before the light turns red while going at the speed limit. Though, this gap should be eliminated in having a longer yellow light time, which will account for the entire GO zone length due to it being a constant. In the end, the goal is to have the yellow light time long enough that the STOP zone and the GO zone are one after another.

With this in mind, we can figure out quickly that the total distance that one must travel, that is the Δx, is going to be the distance of the intersection, x_{int}, plus the distance of the end of the stop zone/start of the GO zone, x_{stop}. Please note x_{stop} is completely unrelated to t_{stop} from prior. This is because the cars all the way at the beginning of the GO zone must make it all the way through the intersection, and that defines the time of the yellow light. Knowing these facts actually give us all of the information we need to make an equation using the $\Delta x = ((v_0 + v) / 2) * t$ kinematic equation. See, we know v_0 is the speed limit, and v is also going to be the speed limit since the velocity is constantly the speed limit while the car is moving through the intersection. We just figured out Δx, so now we can solve for t:

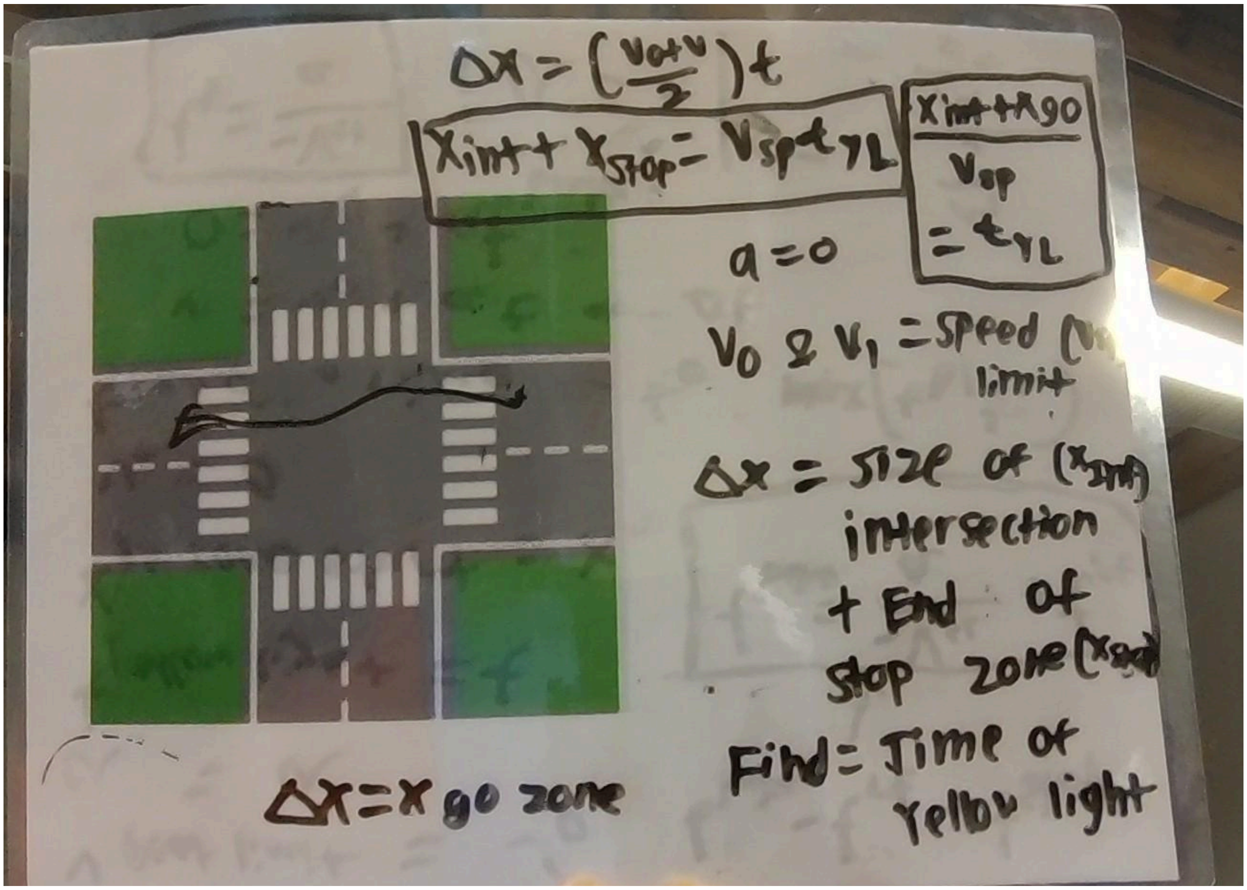
$\Delta x = ((v_0 + v) / 2) * t$

$x_{int} + x_{stop} = ((v_0 + v) / 2) * t$

$x_{int} + x_{stop} = (2v_0 / 2) * t$

$x_{int} + x_{stop} = v_0 * t$

$t = (x_{int} + x_{stop}) / v_0$



Now we have a formula that takes the size of the intersection into consideration when finding the time of the yellow light. In words, our formula says the length of the yellow light should be the total distance traveled by a car at the start of the GO zone divided by the speed limit. But we decided to go a bit further. This formula is great and all, but we wanted to define everything in terms of what we knew, and GO zone and STOP zone distances were not things we knew. So our new goal: rewrite the equation without any references to the GO or STOP zones.

Let's first work towards expressing x_{stop} in terms of other variables. Considering just the distance of the GO zone, we have Δx, which is x_{stop}, along with v_o and v. Earlier, we calculated how long it would take a car to stop just before the intersection, and we can use this same formula to now find the length of the zone. This means v is the speed limit, and v_0 is simply zero. t is obviously the time to stop, but we have further broken that down into its own equation, which we can use here:

$\Delta x = ((v_0 + v) / 2) * t$

$x_{stop} = (v_0 / 2) * t$

$x_{stop} = (v_0 / 2) * (-v_0 / a + t_{rt})$

So the distance of the GO zone is equal to the speed limit divided by two (the average velocity as the car decelerates), times the time it takes the car to stop. Now we can plug this back into the equation we used to find the time of the yellow light in place for x_{stop}:

$x_{int} + x_{stop} = v_0 * t$

$x_{int} + (v_0 / 2) * (-v_0 / a + t_{rt}) = v_0 * t$

$(x_{int} + (v_0 / 2) * (-v_0 / a + t_{rt})) / v_0 = t$

Simplified down the formula looks like this: (which is not on the whiteboard)

$(x_{int} / v_0) + (v_0 / 2) * (-v_0 / a + t_{rt}) / v_0 = t$

$(x_{int} / v_0) + (½)(v_0) * (-v_0 / a + t_{rt}) / v_0 = t$

$(x_{int} / v_0) + (½) * (-v_0 / a + t_{rt}) = t$

Which reads as the time taken to move through the intersection plus half the time it takes a driver to come to a full stop is the time the yellow light needs to be. This is our final solution.

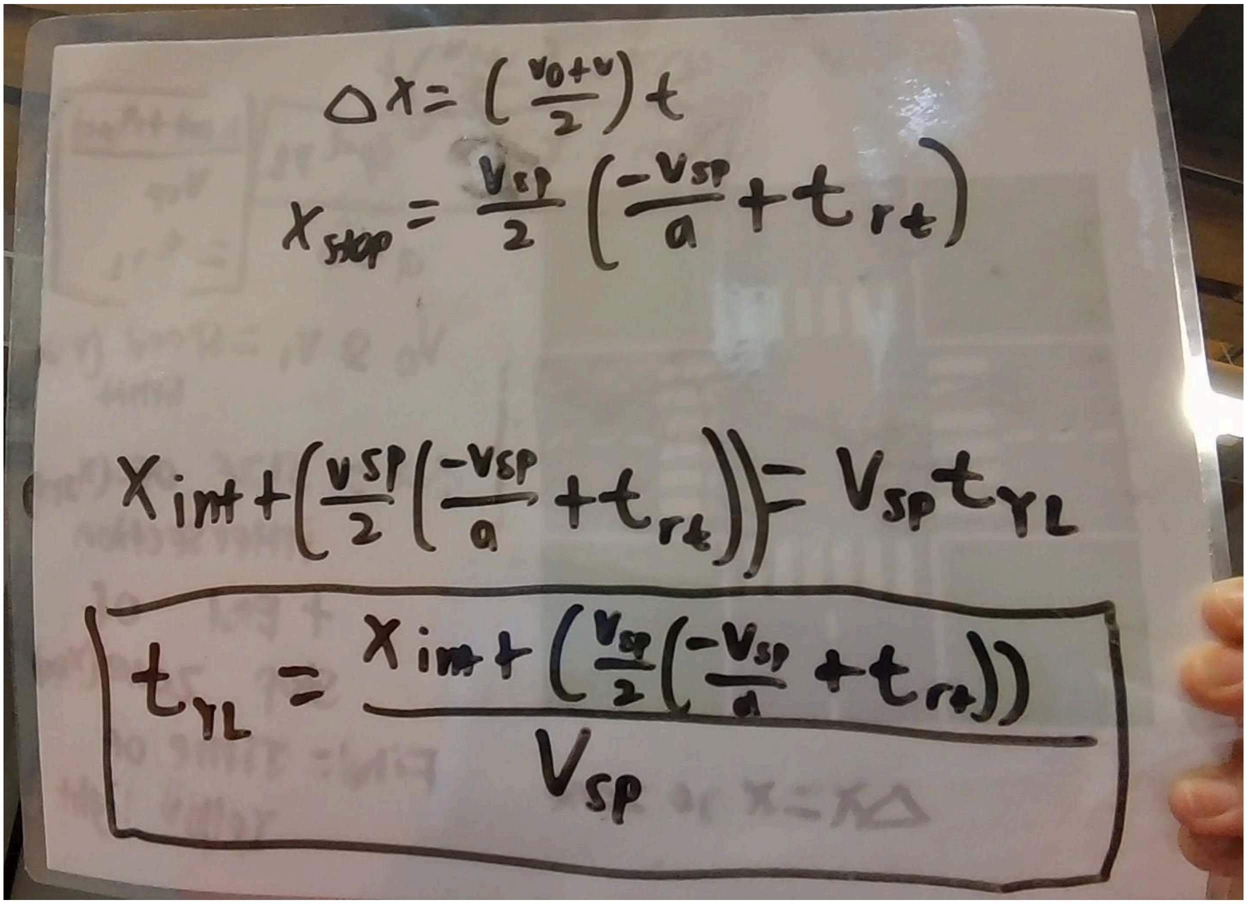
$$\Delta x = \left(\frac{v_0 + v}{2}\right) t$$

$$x_{stop} = \frac{v_{sp}}{2}\left(\frac{-v_{sp}}{a} + t_{rt}\right)$$

$$x_{int} + \left(\frac{v_{sp}}{2}\left(\frac{-v_{sp}}{a} + t_{rt}\right)\right) = v_{sp} t_{YL}$$

$$\boxed{t_{YL} = \frac{x_{int} + \left(\frac{v_{sp}}{2}\left(\frac{-v_{sp}}{a} + t_{rt}\right)\right)}{v_{sp}}}$$

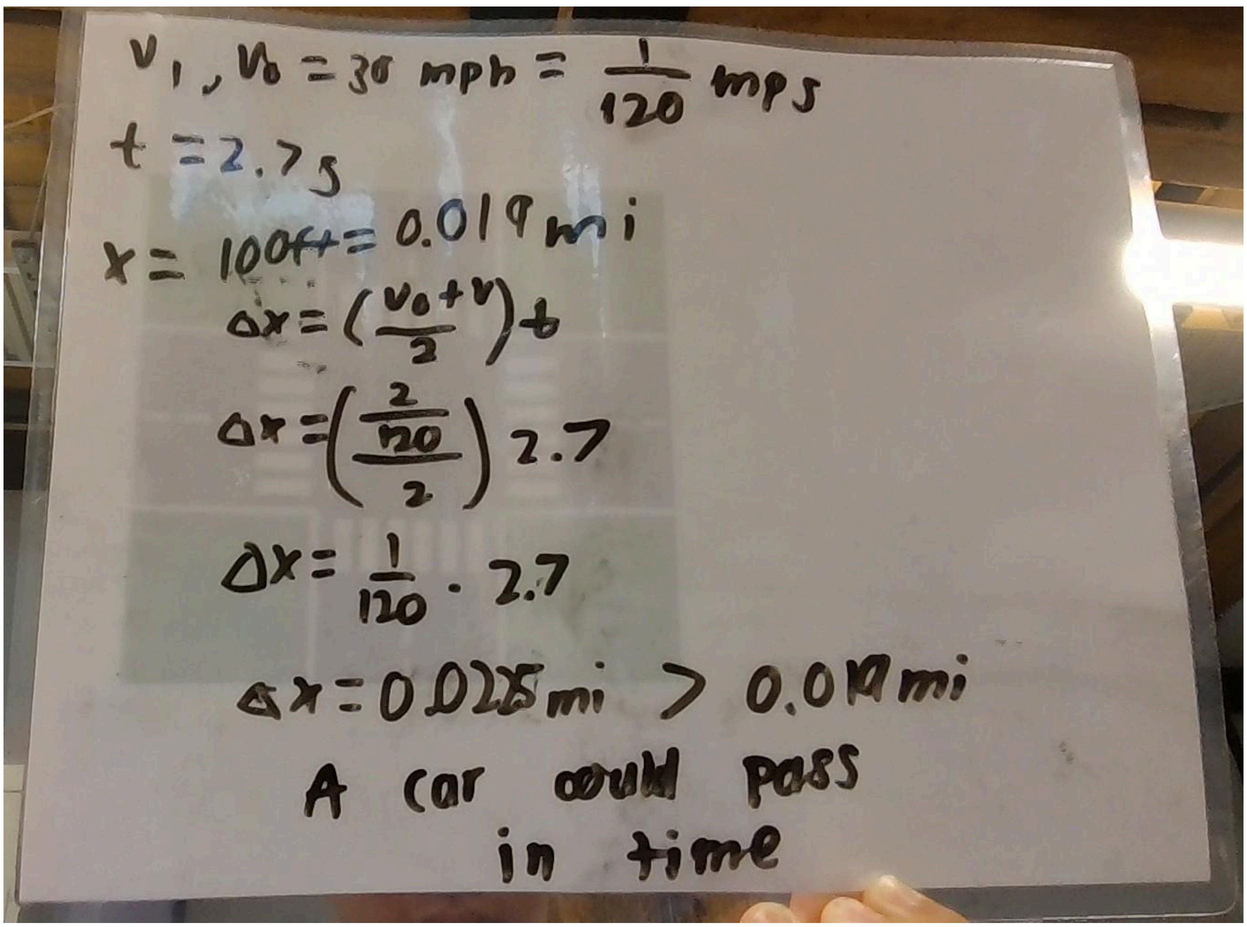
Solution



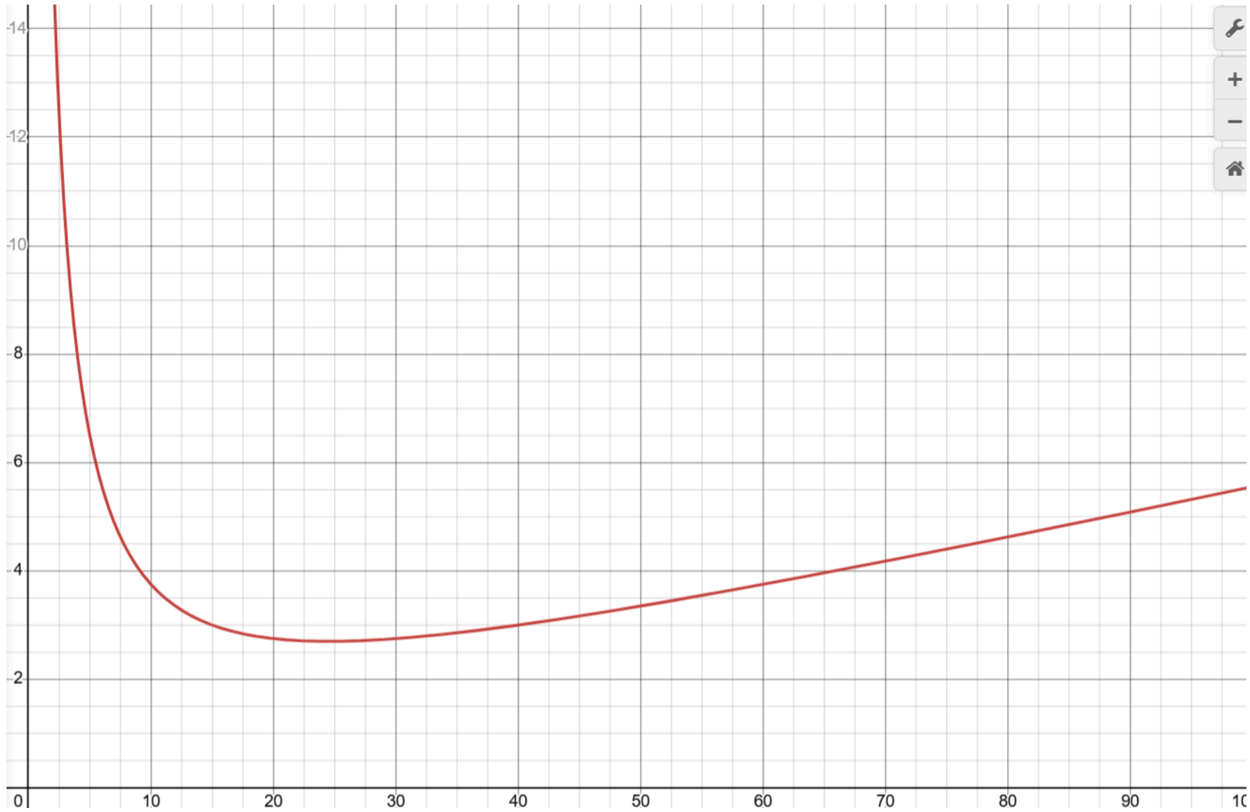
Our final solution is:

Time of yellow light = (Size of intersection + (Speed Limit / 2 (-Speed Limit / Acceleration when breaking + Reaction Time))) / Car Velocity at Speed Limit

We tested our solution in two scenarios: one with a 30MPH speed limit and one with a 60MPH speed limit. We gave an arbitrary number for our acceleration with -10m/s^2, a Reaction Time of one second, and the size of the intersection being 100ft. We made a graph using these values, with the X-axis being the speed limit and the Y-axis being the time of the yellow light. In both of our scenarios, we got values that made sense. (2.7s and 3.73s respectively) They were small but not so small that a car moving at the speed limit wouldn't be able to get across. We then used our time to see how much distance a car could cover in that time and if a car would be able to cross the intersection in time. We found that in both of our scenarios, at least one car could pass when the light turned yellow. We calculated that the cars in the STOP zone would have to stop the moment they see the yellow light.



Math that proves a car moving 30mph could pass through the intersection in time.



Here is a graph of the yellow light time in terms of the speed limit with the variables defined as:

Distance of the intersection is 30m

Deceleration is -10m/s^2

Reaction Time is 0.5s

Discussion Questions

a. What must be true about the STOP and GO zones in order for the intersection to be safe? Describe what happens at the intersection if the zones do not adhere to this rule.

The STOP and GO zones must be either touching or overlapping in a real life scenario, as otherwise there would be a gap between them. This gap would be incredibly unsafe because it would prevent drivers in the gap from being able to cross the intersection entirely and wouldn't give them ample space to slow down to a stop before entering the intersection. This would cause drivers to be in the middle of the intersection when the light turns red, regardless of if they choose to brake or keep driving.

b. How would the following conditions affect the required yellow light time? Which zone would they affect? via which variable?

i. Exceeding the speed limit

Based on our graph, If the speed limit was exceeded, the yellow light time would be decreased until a certain speed where it would begin to increase. The speed at which it begins to increase changes based on the distance of the intersection. This is because at lower speeds the yellow light needs to be longer to ensure that cars make it all the way through the intersection in time, and at higher speeds the cars need less time to make it through the intersection but more time to slow down if the light turns yellow. The line between the GO zone and STOP zone would move accordingly, getting closer with less time and farther with more time.

ii. Bald tires

If the tires were bald that means the car would have less grip on the road so acceleration would be closer to zero and it would take longer to stop. This would affect our acceleration by making it into a value closer to zero. If acceleration is closer to zero then the traffic light would stay yellow for longer because our solution has acceleration in the denominator. This would cause the STOP zone to be smaller because more time is required for the car to come to a full stop. Because the acceleration is smaller, the yellow light time would increase based on our equation, so the GO zone would get bigger because the yellow light would stay yellow for longer.

iii. Long vehicle such as a semi-truck

A longer or larger vehicle like a semi-truck would take longer for the vehicle to stop. In the context of this specific vehicle, its acceleration would be smaller. This would cause the line of the STOP zone to be moved further away from the intersection to show that the truck needs to begin braking earlier. Because acceleration is smaller, the time of the yellow light would increase based on our equation, in turn increasing the GO zone.

iv. Distracted driver

If there is a distracted driver, then their reaction time will be increased. A greater reaction time would cause the driver to take longer to react to the yellow light changing from green to yellow, requiring them more time to come to a complete stop before they enter the intersection. Because the car will take longer to stop, the line of the STOP zone would be moved further back to show that cars will need to stop earlier to comfortably stop before the light. Because it would take longer to stop, the time of the yellow light would increase based on our equation, in turn increasing the size of the GO zone.

c. Would it be a good idea to mark the stop and GO zones on the road before the intersection? Why or why not?

Although in the context of this problem, marking the stop and GO zones won't change anything, in a practical sense it could help drivers know if they are able to make it through the intersection or not which would allow them to not have to guess if it is safe. It might be able to prevent accidents and prevent people from running red lights if implemented. However, we believe that this would cause more harm than benefit as it would also cause people to try to go through the intersection anytime they are in the GO zone despite traffic and other variables. Labelling the STOP zone on the ground would also confuse drivers which could cause some to STOP in the middle of a greenlight. On top of this, due to every vehicle having a separate brake time and everyone having a different reaction speed the GO zone/STOP zone would be separate for everybody.

d. In the 1960's traffic engineers piloted a system that would display a countdown timer to show how much time was left before the light would turn red. Why do you think the engineers decided against the idea?

We think engineers decided against this idea because if people knew how much time was left, they might speed up to try and pass the light in time. By leaving the time out, people would usually choose the safer option, which is to stop, because drivers would not know how much time they had to possibly risk running a red light. Drivers could have also paid too much attention to the timer, choosing to avert their attention from the road to see if they made the timer, this could cause dangers around the driver due to them not paying ample attention to their surroundings.