Happy Birthday POW

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1. Problem Statement:

The seven-day week is used throughout the world, and many cultures have sayings about how the day of the week on which a person is born might affect who they are. That raises the question, though. What day of the week was someone born on?

That is the question we were asked to figure out. We were tasked with creating a system to find the day of the week on which someone was born. All we were given was the date when the person was born. We also only had to worry about people born after 1900 in order to avoid the special rules when the year is a multiple of 100. We had to create a step-by-step solution that was clear and easy to follow so that anyone can find the day of the week they are born on.

2. Process:

First, we tried out some test values such as June 1st and July 4th. We took the number of days away from October 1st (which we knew the day of the week of). Then we used mod 7, which means dividing by 7 and then taking the remainder as the answer (For example 9 mod 7 is 2 because 9/7 = 1 and 2/7, the 2 is the remainder so it is our answer). For June 1st, it was 122 days away, and the mod 7 was 3. Because we were going backwards in time, this meant it was 3 days of the week before the day of the week that October 1st was. On our calendar, October 1st was a Wednesday, so going 3 days back gave us Sunday, meaning June 1st was a Sunday. Next, we used the example of July 4th because, from past experiences, we knew that July 4th was a Friday. July 4th was 89 days away from October 1st, so we took mod 7 of 89, which was 5. We looked at October 1st (Wednesday) and moved 5 days before to conclude that July 4th was on a Friday.

Variables:

- The starting day of the week

- The starting day of the month
- The starting month
- The starting year
- The desired day of the month
- The desired month
- The desired year

Next, we tried a date that was forward in time. We tried out the example of knowing October 1st but not the 23rd. We found that the 23rd is 22 days away from the 1st. Took mod 7 of that, which was 1. Typically, this would mean that we would go 1 day back from October 1st, making the 23rd on a Tuesday, but because we are going forward in time, the real day of the week is one day forward, so it is on a Thursday

Now that we have found a way to determine the day of the week, we need to simplify the process to make it easier than counting every day to a date.

To simplify the process, we can find the mod for every month and add it up that way, so rather than counting the days of every month, someone finding out a desired day of the week could add up the mods of every month separating them to the desired date, add the mod of the day of the month, and subtract the mod of the desired day of the month.

(Going from March 2^{nd} to January 3^{rd} would be the mod of days in February + the mod of days in January + 2 (starting day of the month) - 3 (ending day of the month))

January	31	3
February	28	0

March	31	3
April	30	2
May	31	3
June	30	2
July	31	3
August	31	3
September	30	2
October	31	3
November	30	2
December	31	3

We tried out the July 4th example again. We assumed we would start on October 6th. We added up 6 (Starting day of the month) + 2 (September) + 3 (August) + 3 (July) – 4 (Ending day of the month), which gave us 10. We still have to mod it by 7, which gives us 3. October 6th is a Monday, but 3 days back is Friday, which is the correct answer.

We then thought of some different ideas to create an easier way to add up days in a month. We decided to create a prefix sum table. First, we put the number of days in a month into mod 7 to help us calculate more easily. Then we created this table:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Days in Year	31	28	31	30	31	30	31	31	30	31	30	31

Mod 7	3	0	3	2	3	2	3	3	2	3	2	3
Distance from Jan 1st in mod 7	3	3	6	1	4	6	2	5	0	3	5	1

The table shows the distance from January 1st of each year in mod 7.

Lastly, we had to figure out how to calculate years apart. 365 mod 7 is 1. This means when we go years apart, the day of the week changes by one day. To find this simply subtract the target year from the start year.

However, we must account for when the leap day occurred. (February 29th). To find out how many leap years occurred between the two dates, we simply divided the starting year and the target year by 4 and then subtracted the quotients.

We must round this number down because leap years happen every 4 years, and do not affect the years between. After rounding that gives us the number of years that are leap years. Then, we created a piecewise function to find the amount of leap days that occurred. We know that a leap day will only play an effect if the date is past February 29th so if the known date is after leap day, then we factor it in. Otherwise, we don't.

Finally, we took all these values that we have been tracking throughout the running counter and added them all together and put them into mod 7. This gives us the number of days of the week away from the starting day of the week. Positive means you are going forward that number of days while negative means you are backwards for that number of days.

3. Solution:

Variables:

- The starting day of the week
- A_{day} The starting day of the month
- A_{month} The starting month
- A_{year} The starting year
- B_{day} The desired day of the month
- B_{month}The desired month
- B_{year} The desired year

Let the known date (anchor) be A (Make sure you know the day of the week that this date is on)

Let the target date be B

Keep a running counter that can be negative or positive

To find years apart, calculate: $B_{year} - A_{year}$. Add this to your counter.

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
#	1	2	3	4	5	6	7	8	9	10	11	12

To find leap days, if $A_{month} \geq 3$ calculate: $\left\lfloor \frac{\mathrm{B_{year}}}{4} \right\rfloor - \left\lfloor \frac{A_{year}}{4} \right\rfloor$

(A_{month} for February would be 2, June would be 6, etc.)

Else if $A_{month} \leq 2$ calculate: $\left\lfloor \frac{\mathrm{B_{year}-1}}{4} \right\rfloor - \left\lfloor \frac{\mathrm{A_{year}-1}}{4} \right\rfloor$.

Round the number you get down and then add this to your counter. (so, 2.5 would be 2, 1.25 would be 1, 6.7 would be 6, etc.)

Next, find the day of the year each date falls on. For both your target and known dates, take the days before each month begins, corresponding to the month and add that to the day of the month.

Next, subtract the sum of your anchor date from your target date.

In other words, let the function $P(\text{month})$ equal the days before each month begins in mod 7.

Mod 7 means we take the total amount of days passed before each month begins, divide it by 7 and write down the remainder.

In other words, substitute values into this: $(P(B_{\text{month}}) + B_{\text{day}}) - (P(A_{\text{month}}) + A_{\text{day}})$.

For example, in a normal year: P(February) = 3, P(August) = 2, and P(November) = 3

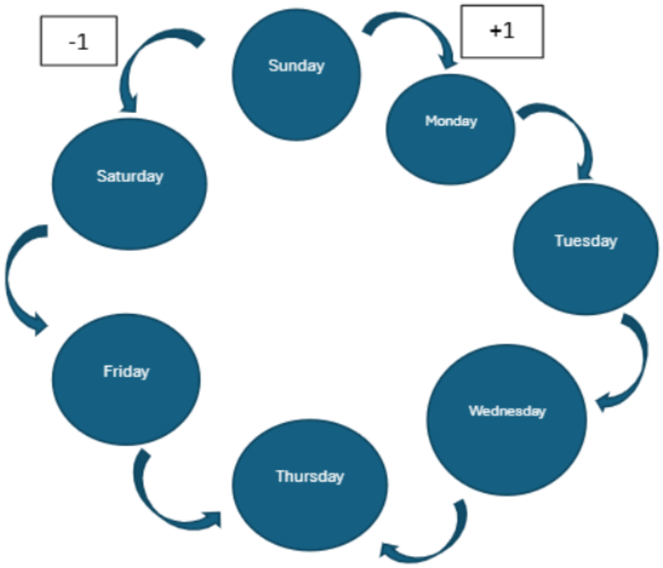
The leap year table will only ever be used if the target year is a leap year and the month is before march or is the starting year is a leap year and the month is after march.

Normal Year												
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Days of previous month	0	31	28	31	30	31	30	31	31	30	31	30
Days before each month begins	0	31	59	90	120	151	181	212	243	273	304	334
Distance from Jan 1st in mod 7	0	3	3	6	1	4	6	2	5	0	3	5

Leap Year												
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Days of previous month	0	31	29	31	30	31	30	31	31	30	31	30
Days before each month begins	0	31	60	91	121	152	182	213	244	274	305	335
Distance from Jan 1st in mod 7	0	3	4	0	2	5	0	3	6	1	4	6

Add your difference to the running counter.

Now divide your final counter by 7 and take the remainder. This is the number of days forward or backwards you need to move in the cycle of days. If the number is positive, move forward with that number of days. If the number is negative, move backwards that number of days.



Formula :

For $A_{month} \geq 3$:

$$\left(B_{year} - A_{year}\right) + \left(\left(\frac{B_{year}}{4}\right) - \left(\frac{A_{year}}{4}\right)\right) + \left(\left(P(B_{month}) + B_{day}\right) - \left(P(A_{month}) + A_{day}\right)\right)$$

For $A_{month} \leq 2$:

$$\left(B_{year} - A_{year}\right) + \left(\left(\frac{B_{year} - 1}{4}\right) - \left(\frac{A_{year} - 1}{4}\right)\right) + \left(\left(P(B_{month}) + B_{day}\right) - \left(P(A_{month}) + A_{day}\right)\right)$$

Where the $\left(\left(\frac{B_{year}}{4}\right) - \left(\frac{A_{year}}{4}\right)\right)$ and $\left(\left(\frac{B_{year}-1}{4}\right) - \left(\frac{A_{year}-1}{4}\right)\right)$ are whole integers rounded down

Then, the final result is divided by seven and the days of the week away from the starting day of the week is the remainder.

Example:

Say we use anchor date of October 1st, 2025 being a Wednesday.

Our target date is November 8th, 2008

First we must find the number of years between the two dates (2008 – 2025 = -17)

We add -17 to our running counter

Now we must find the amount of leap days. Since November (11) is greater than 3 we can use this formula: $\left\lfloor\frac{\mathrm{B_{year}}}{4}\right\rfloor - \left\lfloor\frac{A_{year}}{4}\right\rfloor$

By plugging in values : $\left\lfloor\frac{2008}{4}\right\rfloor - \left\lfloor\frac{2025}{4}\right\rfloor$ we get -4.25 but must round down to -4

We must add this to our running counter giving us -21

Next, we must find the number of days passed in one year.

We can use this formula: $(P(B_{\mathrm{month}}) + B_{\mathrm{day}}) - (P(A_{\mathrm{month}}) + A_{\mathrm{day}})$.

If we plug in values, we get: $(P(November) + 8) - (P(October) + 1)$.

If we use the table to find the P we get: $(3 + 8) - (0 + 1)$. Giving us 10

We add this to our running counter: -11

Now we have to divide by 7 and find the remainder which is -4

This means that we must go 4 days of the week back from Wednesday. This gives us Sunday, which is the correct answer.

4. Extensions:

The first extension we thought of was to increase the range of our formula to account for dates before 1900 and after the current year. We thought it would be very interesting because the rules to account for in a formula increase as you try to predict dates further and further back.

- To account for dates before 1900, we would need to consider the rule that leap years are skipped every 100 years, except for years that are multiples of 400, meaning 1900 would not be a leap year, but 2000 would.
- We can assume the calendar stays consistent, so to predict forward in time, we could use the same process, but adding 1 for every year instead of subtracting up until the year 2100. At 2100, the rule stating that there is not a leap year in a year that is a multiple of 100 comes into play.

The second extension we thought of was to create a formula to predict the day of the week someone was born on for a different calendar. There are many calendars used around the world, so it would be interesting to create different formulas to account for the changes in calendars.

- Some calendars make a formula easier to create, such as The Ethiopian Calendar, which has 13 months. There are 12 months with 30 days and the final month changes between 5 and 6 days depending on if it is a leap year, for this calendar the formula could become much simpler because the days between every month is standardized, aside from the last month which could be accounted for without too much trouble by considering it in the year's total and calculating it separately from the other 12 months when predicting a day of the week within that month.
- There are also some other calendars that make a formula much more difficult to create, such as the Buddhist calendar, which is based on a lunar month, making every month take either 29 or 30 days. This causes a common year to have around 354 days, but to account for a solar year, there are leap years, which add an entire month where the number of days is completely based on another lunar month, varying in length, making the whole year during a leap year vary between 384 to 385 days.