Ma2201/CS2022

Discrete Mathematics

D Term, 2019

Quiz 0111

Print Name: ____________________

Sign: ____________________

☐ Ordinary Quiz ☐ Double Down ☐ Triple Down

1. (**6 points**) Let $|X| = 2$ and $|Y| = 7$, $Z = \mathbb{N} \times \mathbb{N}$.

Label each of the following T if it must be true, F if it must be false and X if it cannot be determined from the information given.

___ $\mathcal{P}(X) \cup \mathcal{P}(Y) \subseteq \mathcal{P}(X \cup Y)$

___ The number of onto functions with domain $\mathcal{P}_6(Y)$ and target $X \times \mathcal{P}_2 X$ is

$$\binom{4}{0}4^6 - \binom{4}{1}3^6 + \binom{4}{2}2^6 - \binom{4}{1}1^6 + \binom{4}{0}0^6$$

___ $\{\emptyset\} \subseteq \mathcal{P}(X) \cap \mathcal{P}(Y) \cap \mathcal{P}(X \times Y)$

___ $\emptyset \in \mathcal{P}(X) \times \mathcal{P}(Y) \times \mathcal{P}(X \times Y)$

___ The relation R, with $R \subseteq [\mathcal{P}(\mathcal{P}(\emptyset)) \cup \mathcal{P}(X \cap Y)] \times [\mathcal{P}(\mathcal{P}(\emptyset)) \cup \mathcal{P}(X \cap Y)]$ defined by $(P, Q) \in R$ if and only if $P \subseteq Q$ is transitive.

___ The set of all functions with domain Y and target $\mathcal{P}(Z)$ is uncountable.

♣ *First is TRUE. Every subset of X is a subset of $X \cup Y$ and the same for the subsets of Y.*

Second is FALSE. It has a couple mistakes. First $|\mathcal{P}_6(Y)| = \binom{7}{6} = 7$, not 6, and $|\mathcal{P}_2(X)| = \binom{2}{2} = 1$, not 2, ($\mathcal{P}_2(X) = \{X\}$), so $|X \times \mathcal{P}_2 X| = 2 \cdot 1 = 2$, not 4. Note, if the target had had cardinality 4, the binomial coefficients would have been fine since $\binom{n}{k} = \binom{n}{n-k}$.

Third is TRUE . $\emptyset$ is a subset of every set, so it is an element of every power set.

Forth is FALSE. $\emptyset$ is not of the form (a, b, c).
It is true that $(\emptyset, \emptyset, \emptyset) \in \mathcal{P}(X) \times \mathcal{P}(Y) \times \mathcal{P}(X \times Y)$.

Fifth is TRUE. The subset $\subseteq$ relation is transitive on any set of sets.

Sixth is TRUE. The domain is finite and the target $\mathcal{P}(\mathbb{N})$ is uncountable. So the just the set of constant functions is uncountable. Or you could reason, even if the domain were just a single element, the set of all functions to an uncountable set would be uncountable.

♣

2. (**4 points**) a) Define a function f with domain $\mathbb{N}$ and target $\mathbb{N} \times \mathbb{N}$ which is one-to-one but not onto.

b) What does the existence of this function allow you to conclude about $|\mathbb{N}|$ and $|\mathbb{N} \times \mathbb{N}|$?

♣ *It is easy to make one-to-one functions on these sets. The onto ones are the ones that are tricky. So the quickest way is to define any one-to-one function and then look to make sure there is a target value not taken, since it will probably be not onto.*

Why not define $f(n) = (n, 0)$. The f is one to one, since each target value (n, m) is related to 0 values if $m \neq 0$, and exactly one value, n if $m = 0$.

It is not onto since target value $(1, 1)$ is not related to domain value.

What a one-to-one function implies is that the cardinality of the domain does not exceed that of the target, so $|\mathbb{N}| \leq |\mathbb{N} \times \mathbb{N}|$. Of course it is not surprising that we get such a weak result from such an easy function. ♣