Ma2201/CS2022

# Discrete Mathematics

D Term, 2019

Quiz 0101

Print Name: ____________________

Sign: ____________________

1. (**6 points**) Define a function $f : \mathbb{N} \times (\mathbb{N} \times \mathbb{N}) \to \mathbb{N} \times \mathbb{N}$ which is onto.
   Explain briefly why your function is onto.
   March a check next to each of the following which you can conclude from the existence of this function.
   ___ $\mathbb{N} \times \mathbb{N} \subseteq \mathbb{N} \times (\mathbb{N} \times \mathbb{N})$
   ___ $|\mathbb{N} \times (\mathbb{N} \times \mathbb{N})| > |\mathbb{N} \times \mathbb{N}|$
   ___ $|\mathbb{N} \times (\mathbb{N} \times \mathbb{N})| \leq |\mathbb{N} \times \mathbb{N}|$
   ___ $\mathbb{N} \times (\mathbb{N} \times \mathbb{N})$ is uncountable.

♣ *There are many many ways of doing this. The elements of* $\mathbb{N} \times (\mathbb{N} \times \mathbb{N})$ *are of the form* $(i, (j, k))$ *where* $i$, $j$, *and* $k$ *are natural numbers. So, say, define* $f(i, (j, k))) = (i, j + k)$.

*This is onto since, given* $\{a, b\} \in \mathbb{N} \times \mathbb{N}$, $f((a, (b, 0))) = (a, b + 0) = (a, b)$.

*The conclusion about cardinality should be that, since the function is onto, the cardinality of the target cannot exceed the cardinality of the domain.* $|\mathbb{N} \times (\mathbb{N} \times \mathbb{N})| \geq |\mathbb{N} \times \mathbb{N}|$

*Of the four statements, only the third one is true, and that cannot be concluded from the onto function we have, since the inequality goes the wrong way. So none of the candidates should be checked.*

2. (**4 points**) For each of the following, label the set as "finite", "countably infinite", "uncountable", or "indeterminant".

   ___ $\{n^2 \mid n \in \mathbb{N}\}$

   ___ $\{q^2 \mid q \in \mathbb{Q}\}$

   ___ The set of all functions with domain $\mathbb{Z}$ and target $\{-1, 0, 1\}$

   ___ $\mathbb{N} \times \mathbb{Q}$.

♣ *We know* $\mathbb{N}$, $\mathbb{Z}$ *and* $\mathbb{Q}$ *are countably infinite. The first two are subsets of countably infinite sets, hence countable.*

*For the third, each function involves a choice, for each element of* $\mathbb{N}$, *of one of three values. That is an infinite number of non-trivial independent choices, so the collection of all those functions is uncountable.*

*For the last, the product of two countably infinite sets is countably infinite.* ♣