Ma2201/CS2022 **Discrete Mathematics** D Term, 2019

Quiz 0100

Print Name: ____________________

Sign: ____________________

1. (**6 points**) How many strings on 12 characters satisfy at least one of the following conditions; they repeat every three characters, like *askaskaskask*; they repeat every 4 characters, like *dripdripdrip*; or the have the second, fifth, eighth, and eleventh character a vowel, ($\{a, e, i, o, u\}$), like *poptoptictac*.

Your answer can be in the form of an algebraic expression, but make sure you don't use any variables you don't define.

♣ *There are* 26^{12} *strings on* 12 *characters, so best to use an algebraic expression.*

Define A *to be the set of strings on 12 characters which repeat every three characters.*

Define B *to be the set of strings on 12 characters which repeat every 4 characters.*

Define C *to be the set of strings on 12 characters which have the second, fifth, eighth, and eleventh character a vowel.*

Using the multiplicative principle, we can compute: $|A| = 26^3$, $|B| = 26^4$, $|C| = 26^8 \cdot 5^4$, $|A \cap B| = 26$, *since all the letters must be the same,* $|A \cap C| = 26^2 \cdot 5$, $|B \cap C| = 5^4$, *since all the characters must be vowels, and and* $|A \cap B \cap C| = 5$ *since all the characters must be equal and vowels.*

So

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ &= 26^3 + 26^4 + 26^8 \cdot 5^4 - 26 - 26^2 \cdot 5 - 5^4 + 5 \qquad ♣ \end{aligned}$$

2. (**4 points**) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let $B = \mathcal{P}_2(\{0, 1\} \times \{0, 2\})$.

a) How many functions are there, $A \to B$.

b) How many functions are there, $A \to B$, which are one to one?

c) How many functions are there, $A \to B$, which are onto?

d) Draw the relation diagram of any one-to-one and onto function with domain and target **both** A, so $A \to A$. Label your diagram clearly and neatly.

(You may use either style diagram. Use the back if you like.)

♣ *We need only note the cardinalities of* A *and* B *for the first three parts.* $|A| = 10$, *and* $|B| = \binom{4}{2} = 6$. *So there are* 6^{10} *functions. None are one-to-one because* B *is too small. For the number of onto functions we need to decorate the 6'th row of Pascal's triangle:*

$$\binom{6}{0} \cdot 6^{10} - \binom{6}{1} \cdot 5^{10} + \binom{6}{2} \cdot 4^{10} - \binom{6}{3} \cdot 3^{10} + \binom{6}{4} \cdot 2^{10} - \binom{6}{5} \cdot 1^{10} + \binom{6}{6} \cdot 0^{10}$$

or

$$1 \cdot 6^{10} - 6 \cdot 5^{10} + 15 \cdot 4^{10} - 20 \cdot 3^{10} + 15 \cdot 2^{10} - 6 \cdot 1^{10} + 1 \cdot 0^{10}$$

are both correct.

For the last part

$$\begin{array}{cccccccccc} \bullet^0 & \bullet^1 & \bullet^2 & \bullet^3 & \bullet^4 & \bullet^5 & \bullet^6 & \bullet^7 & \bullet^8 & \bullet^9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \bullet^0 & \bullet^1 & \bullet^2 & \bullet^3 & \bullet^4 & \bullet^5 & \bullet^6 & \bullet^7 & \bullet^8 & \bullet^9 \end{array}$$

works for the identity function, which is one to one, or for something different in the other style:

$$\begin{array}{ccccccccc} \bullet^0 & \rightarrow & \bullet^1 & \rightarrow & \bullet^2 & \rightarrow & \bullet^3 & & \bullet^4 \\ \uparrow & & & & & & \downarrow & & \updownarrow \\ \bullet^9 & \leftarrow & \bullet^8 & \leftarrow & \bullet^7 & \leftarrow & \bullet^6 & & \bullet^5 \end{array}$$

♣