



1. (**6 points**) How many strings on 12 characters satisfy at least one of the following conditions; they repeat every three characters, like *askaskaskask*; they repeat every 4 characters, like *dripdripdrip*; or the have the second, fifth, eighth, and eleventh character a vowel, ($\{a, e, i, o, u\}$), like *poptoptictac*.

Your answer can be in the form of an algebraic expression, but make sure you don't use any variables you don't define.

♣ *There are 26^{12} strings on 12 characters, so best to use an algebraic expression.*

Define A to be the set of strings on 12 characters which repeat every three characters.

Define B to be the set of strings on 12 characters which repeat every 4 characters.

Define C to be the set of strings on 12 characters which have the second, fifth, eighth, and eleventh character a vowel.

Using the multiplicative principle, we can compute: $|A| = 26^3$, $|B| = 26^4$, $|C| = 26^8 \cdot 5^4$, $|A \cap B| = 26$, since all the letters must be the same, $|A \cap C| = 26^2 \cdot 5$, $|B \cap C| = 5^4$, since all the characters must be vowels, and $|A \cap B \cap C| = 5$ since all the characters must be equal and vowels.

So

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ &= 26^3 + 26^4 + 26^8 \cdot 5^4 - 26 - 26^2 \cdot 5 - 5^4 + 5 \quad \clubsuit \end{aligned}$$

2. (**4 points**) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let $B = \mathcal{P}_2(\{0, 1\} \times \{0, 2\})$.

a) How many functions are there, $A \rightarrow B$.

b) How many functions are there, $A \rightarrow B$, which are one to one?

c) How many functions are there, $A \rightarrow B$, which are onto?

d) Draw the relation diagram of any one-to-one and onto function with domain and target **both** A , so $A \rightarrow A$. Label your diagram clearly and neatly.

(You may use either style diagram. Use the back if you like.)

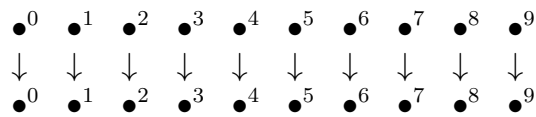
♣ *We need only note the cardinalities of A and B for the first three parts. $|A| = 10$, and $|B| = \binom{4}{2} = 6$. So there are 6^{10} functions. None are one-to-one because B is too small. For the number of onto functions we need to decorate the 6'th row of Pascal's triangle:*

$$\binom{6}{0} \cdot 6^{10} - \binom{6}{1} \cdot 5^{10} + \binom{6}{2} \cdot 4^{10} - \binom{6}{3} \cdot 3^{10} + \binom{6}{4} \cdot 2^{10} - \binom{6}{5} \cdot 1^{10} + \binom{6}{6} \cdot 0^{10}$$

or

$$1 \cdot 6^{10} - 6 \cdot 5^{10} + 15 \cdot 4^{10} - 20 \cdot 3^{10} + 15 \cdot 2^{10} - 6 \cdot 1^{10} + 1 \cdot 0^{10}$$

are both correct.
 For the last part



works for the identity function, which is one to one, or for something different in the other style:

