Ma2201/CS2022 **Discrete Mathematics** D Term, 2019

Quiz 0011

Print Name: ____________________

Sign: ____________________

1. (**7 points**) Prove carefully using the double inclusion method that

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C).$$

♣ *We first show $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$. Let $x \in (A \cap B) \cup C$. So $x \in A \cap B$ or $x \in C$.*

Case 1: If $x \in A \cap B$, then $x \in A$ and $x \in B$. So $x \in A \cup C$ and $x \in B \cup C$. Thus $x \in (A \cup C) \cap (B \cup C)$.

Case 2: If $x \in C$, then $x \in A \cup C$ and $x \in B \cup C$. Thus $x \in (A \cup C) \cap (B \cup C)$ in this case as well.

Therefore $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$.

Next we show that $(A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C$. Let $y \in (A \cup C) \cap (B \cup C)$. So $y \in A \cup C$ and $y \in B \cup C$. (Here the first fact gives two cases and the second fact two as well, so there are four cases altogether to be considered, but we can get away with just two.) There are two cases, either $y \in C$ or $y \not inC$.

Case 1: $y \in C$, then $y \in (A \cap B) \cup C$.

Case 1: $y \notin C$, then since $y \in (A \cup C)$, we have $y \in A$. And since $y \in (B \cup C)$, we have $y \in B$. So $y \in A \cap B$, and thus $y \in (A \cap B) \cup C$.

Therefore $(A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C$.

Since $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$. and $(A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C$. we have that the sets are equal.

♣

2. (**3 points**) Let A and B be sets with $A = \{1, 2, 3, 4\}$ and $|B| = 2^{|A|}$, and define X by $X = \mathcal{P}(A \cap B) \times \mathcal{P}(\mathcal{P}(\emptyset))$.

For **three** of the following, label the statement by T if it must be true, F if it must be false, and X if it cannot be determined by the information given.

__ $X \subseteq \mathcal{P}(A \times \mathcal{P}(\emptyset))$.

__ $X = \emptyset$.

__ $|X| \leq 2^6$.

__ $2^4 \leq |X \times \mathcal{P}(X)|$.

♣ *$FFTX$ — This is very simple if you know the basic results.*

For the first one, the elements of X are pairs, while, on the other hand, the elements of $\mathcal{P}(A \times \mathcal{P}(\emptyset))$ are not pairs, but sets of pairs. (Like $\{(1, \emptyset), (2, \emptyset)\}$). So regardless of their actual form X cannot be subset unless it is empty, which it isn't since $(\emptyset, \emptyset) \in X$.

The set B is rather big, and we didn't specify its contents, but it's intersection with A can have at most 4 elements, so $0 \leq |A \cap B| \leq 4$, so $2^0 \leq |\mathcal{P}(A \cap B)| \leq 2^4$.

For the other factor, $|\mathcal{P}(\mathcal{P}(\emptyset))| = 2^{(2^0)} = 2^1 = 2$, so $2 \leq |X| \leq 2^5$, for the last bit $2 \cdot 2^2 \leq |X \times \mathcal{P}(X)| \leq 2^5 \cdot 2^{(2^5)}$. ♣