1. (**4 points**) A magician designs a value guessing trick with 5 cards, but with the question for each card being:

"Is the secret number on this card and on at least one of the other cards?"

Label each of the following TRUE if it must be true, FALSE if it must be false, and X if it cannot be determined from the information given.

__ The number of values distinguished can be computed by the multiplicative principle.

__ The answers are not independent.

__ The largest number of values m that can be distinguished satisfies $m \leq 32$.

__ The largest number of values m that can be distinguished satisfies $m < 32$.

♣ *A. F — The answers are dependent, (you cannot get YNNNN) so the multiplicative principle only gives an upper bound.*
B. T — The answers are dependent.
C. T — True by multiplicative principle.
D. T — Also true by the multiplicative principle, since the questions are dependent, $2^5 = 32$ cannot be achieved. ♣

2. (**4 points**) A website requires you to pick an eight character password, using the letters a, b, ... z, like zeezypop, and to pick a four digit pin, like 0254. To make it easier to remember, many people choose the first four characters of their password to be the initial letters of the digits in their pin.

So if their pin is 6789, their password would start ssen....

How many passwords using this system are there?

If everybody used this system, would it be more or less secure to have them just use a password with four digits followed by four letters, or just the same?

♣ *The digits* **z***ero,* **o***ne,* **t***wo,* **t***hree,* **f***our,* **f***ive,* **s***ix,* **s***even,* **e***ight, or* **n***ine have 7 initial letters z, o, t, f, s, e, or n. So there are 7 possibilities for each of the first four characters, and 26 for the last four. The choices are independent, and there are $7^4 \cdot 26^4$.*

There would be more passwords if you had four digits and four letters: $10^4 \cdot 26^4$. ♣

3. (**2 points**) Suppose your sister has had a baby and you want to congratulate her. Of the following methods, circle the one you consider most discrete, and write a word or two why.

Underline the one you consider the least discrete, and write a word or two why.

– Send her an mp3 of you singing a jolly song.

– Send her a text message over Skype with a cute dancing emoticon.

– Send her congratulations in Morse code.

– Scream "CONGRATULATIONS!" while she is rocking the sleeping baby.

♣ *It is easy to justify that the second and third are discrete. The last seems particularly indiscrete to me.* ♣