Ma2201/CS2022 Quiz 0011

Discrete Mathematics

D Term, 2018

Print Name: ____________________

Sign: ____________________

1. (**6 points**) Let A and B be sets. Prove carefully by the double inclusion method that

$$A^c \cap B^c = (A \cup B)^c.$$

♣ *Remember, a proof is a convincing argument. A list of equations is not an argument.*

First we prove $A^c \cap B^c \subseteq (A \cup B)^c$. *Let* $x \in A^c \cap B^c$. *So* $x \in A^c$ *and* $x \in B^c$. *So* $x \notin A$ *and* $x \notin B$, *hence* $x \notin A \cup B$. *So* $x \in (A \cup B)^c$, *so* $A^c \cap B^c \subseteq (A \cup B)^c$.

Next we prove $(A \cup B)^c \subseteq A^c \cap B^c$. *Let* $y \in (A \cup B)^c$, *so* $y \notin A \cup B$. *So* $y \notin A$ *and* $y \notin B$, *in other words,* $y \in A^c$ *and* $y \in B^c$. *Thus* $y \in A^c \cap B^c$. *Therefore* $(A \cup B)^c \subseteq A^c \cap B^c$.

Since $A^c \cap B^c \subseteq (A \cup B)^c$ *and* $(A \cup B)^c \subseteq A^c \cap B^c$, *we have* $A^c \cap B^c = (A \cup B)^c$.

♣

2. (**4 points**) How many numbers in the range of at least one million but less than two million are either even, divisible by 5, or have the same last three digits.

Justify your answer.

These are all seven digit numbers but the first digit is always 1. Let X *be the set of such numbers which are even,* Y *be the set which are divisible by* 5, *and* Z *be those with the last three digits equal.*

$|X| = 10^5 \cdot 5$ – *last digit is in* $\{0, 2, 4, 6, 8\}$.

$|Y| = 10^5 \cdot 2$ – *last digit is in* $\{0, 5\}$.

$|Z| = 10^4$ – *last two digits forced.*

$|X \cap Y| = 10^5$ – *last digit is* 0.

$|X \cap Z| = 10^3 \cdot 5$ – *last three digits are equal and in* $\{0, 5\}$.

$|Y \cap Z| = 10^3 \cdot 2$ – *last three digits are equal and in* $\{0, 2\}$

$|X \cap Y \cap Z| = 10^3$ – *last three digits equal and* 0.

So

$$\begin{aligned} |X \cup Y \cup Z| &= |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z| \\ &= 10^5 \cdot 5 + 10^5 \cdot 2 + 10^4 - 10^5 - 10^3 \cdot 5 - 10^3 \cdot 2 + 10^3 \end{aligned}$$