Ma2201/CS2022 **Discrete Mathematics** D Term, 2018

Quiz 1111

Print Name: ____________________

Sign: ____________________

1. (**8 points**) Let $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$. Compute each of the following.
   You may you expressions like 5^7, $\binom{7}{5}$, or $\genfrac\{\}{0pt}{}{7}{5}$ in you answers.
   a) How many functions are there with domain $\mathcal{P}(A)$ and target $\mathcal{P}(\mathcal{P}(B))$?

$\clubsuit$ *By the multiplicative principle, the domain has cardinality* $2^3 = 8$ *and the target has cardinality* $2^8 = 256$. *So the number of functions is* $(2^8)^{(2^3)} = 256^8$. $\clubsuit$

b) How many onto functions are there with domain $\mathcal{P}(B)$ and target $\mathcal{P}(A)$?

$\clubsuit$ *Both the domain and target have cardinality* 8, *so the onto functions are in fact one-to-one and onto, so there are* 8! *of them.*

That is equivalenet to $\genfrac\{\}{0pt}{}{8}{8}8!$ *or we can use the 8'th row of Pascal's triangle and modify with alternating 8'th powers,*

$$\binom{8}{8}8^8 - \binom{8}{7}7^8 + \binom{8}{6}6^8 - \binom{8}{5}5^8 + \binom{8}{4}4^8 - \binom{8}{3}3^8 + \binom{8}{2}2^8 - \binom{8}{1}1^8 + \binom{8}{0}0^8$$

$\clubsuit$

c) How many onto functions are there with domain $\mathcal{P}(B)$ and target A?

$\clubsuit$ *The target has cardinality 3, which is less than the cardinality, 8, of the domain, so there are onto functions.*

Now we must use $\genfrac\{\}{0pt}{}{8}{3}3!$ *or the inclusion exclusion formula, but there are many fewer terms:*

$$\binom{3}{3}3^8 - \binom{3}{2}2^8 + \binom{3}{1}1^8 - \binom{3}{0}0^8$$

$\clubsuit$

d) How many onto functions are there with domain $\mathcal{P}(B)$ and target A if the elements of the domain are are regarded as indistinguishable?

$\clubsuit$ *Recalling the Santa Claus problem, this is the "dollars to children" version.*

The number is $\binom{8-1}{3-1} = \binom{7}{2}$. $\clubsuit$

2. (**2 points**) Let X be a set and suppose $f : \mathbb{N} \to X$ is one-to-one and $g : \mathcal{P}(X) \to \mathbb{N}$ is onto. Label the following T if it must be true,. F if it must be false, and X if no conclusion can be drawn.

___ X is countably infinite.

___ $\mathcal{P}(X)$ is countably infinite.

$\clubsuit$ *From* f *we conclude* $|\mathbb{N}| \leq |X|$, *so* X *is an infinite set, perhaps countable, perhaps not. That's enough for us to conclude that* $\mathcal{P}(X)$ *is uncountable, since it is the power set of an infinite set. From* g *we conclude* $|\mathcal{P}(X)| \geq |\mathbb{N}|$, *be we already know* $|\mathcal{P}(X)| > |\mathbb{N}|$ *from* f. *So the first statement is unknown,* X, *and the second is false,* F. $\clubsuit$