



Ma2201/CS2022

Quiz 0110 – DOUBLE DOWN

# Discrete Mathematics

D Term, 2018

PRINT NAME: \_\_\_\_\_

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Ordinary Quiz     Double Down     Triple Down

1. (3 pts) How many numbers in the range of at least one million but less than two million either start with two odd digits, or end in three odd digits, or have all equal digits.

**Justify your answer.**

## Solution

Let  $A$  be the set which starts with 2 odd digits,  $B$  be those that end in three odd digits, and  $C$  be those with all equal digits, so that we want  $|A \cup B \cup C|$ , and use inclusion/exclusion. Notice that the first of the seven digits must be 1, so if all the digits are equal it must be 1, 111, 111.

$$|A| = 5 \cdot 10^5, |B| = 5^3 \cdot 10^3, |C| = 1.$$

$$|A \cap B| = 5^4 \cdot 10^2, |A \cap C| = 1, |B \cap C| = 1.$$

$$|A \cap B \cap C| = 1$$

So the result is

$$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = 5 \cdot 10^5 + 5^3 \cdot 10^3 + 1 - 5^4 \cdot 10^2 - 1 - 1 + 1$$

(You could also notice that  $C \supseteq A$  and  $C \subseteq B$ , so in a fast solution would be two ignore  $C$ , and indeed, all the terms with  $C$  cancel out.)

2. (3 pts) Let  $A$  and  $B$  be sets with with  $A \subset B$ .

Label each of following as  $T$  if it must be true,  $F$  if it must be false, or  $X$  if it cannot be determined from the information given.

\_\_\_  $A \in \mathcal{P}(B)$

\_\_\_  $\emptyset \in (\mathcal{P}(A) \cap \mathcal{P}(B))$

\_\_\_  $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$

## Solution

These are all  $T$ .  $A$  is a subset of  $B$  so it is *an element of* the power set. The empty set is a subset of every set, so it *an element of* every power set, so it is in the intersection. Lastly, every subset of  $A \cap B$  is a subset of  $A$ , and a subset of  $B$ .

3. (4 pts) Suppose for  $n \in \mathbb{N}$  that  $p_n$  is a statement, and suppose  $p_k \Rightarrow p_{k+2}$  for all  $k \in \mathbb{N}$ . Suppose also that  $p_{1234} \wedge \neg p_{4321}$  is true.

For each of the following, label the statement  $T$  if it must be true,  $F$  if it must be false, and  $X$  if it cannot be determined.

\_\_\_  $p_{12345}$

\_\_\_  $p_{12344}$

\_\_\_  $p_{123}$

\_\_\_  $p_{12}$

### Solution

The inductive statements relate the even and odd statements. Since  $p_{1234}$  is true, every statement with index even and larger than 1234 is true by induction. Since  $p_{4321}$  is false, no statement with odd index less than 4321 can be true, otherwise  $p_{4321}$  would be true by induction.

So the truth of  $p_{12345}$  cannot be determined,  $X$ ;  $p_{12344}$  must be true,  $T$ ;  $p_{123}$  must be false  $F$ , and  $p_{12}$  is unknown,  $X$ .