Ma2201/CS2022 **Discrete Mathematics** D Term, 2018
Quiz 0101

PRINT NAME: _____
SIGN: _____

1. (6 pts) Prove carefully by induction that

$$(1.1)^n \geq \frac{10+n}{10}$$

for all $n \geq 1$.

We prove this by induction on n.

For the base case let $n = 1$, so the statement is $1.1 \geq 11/10$ which is true since the numbers are equal.

For the induction step, let $(1.1)^n \geq (10+n)/10$ for some particular n.

Consider $(1.1)^{n+1}$.

$$\begin{aligned} 1.1^{n+1} &= 1.1^n \cdot 1.1 \\ &\geq \frac{10+n}{10} \cdot (1.1) \quad \text{by the induction hypothesis} \\ &= \frac{11+1.1n}{10} \\ &> \frac{11+n}{10} \quad \text{since } 1.1n > n \\ &= \frac{10+(n+1)}{10} \end{aligned}$$

as required for the statement for $n+1$.

So the result is true for all $n \geq 1$ by induction.

2. (4 pts) Suppose for $n \in \mathbb{N}$ that p_n is a statement, and suppose $p_k \Rightarrow p_{k+10}$ for all $k \in \mathbb{N}$. Suppose also that $p_{55} \wedge \neg p_{52}$ is true.

For each of the following, label the statement T if it must be true, F if it must be false, and X if it cannot be determined.

____ $p_{102030405}$
____ $p_{504030201}$
____ p_1
____ p_2

First statement T, by induction with p_{55} being the base case, and the $p_k \Rightarrow p_{k+10}$ being the inductive step, so the statement is true for those statements p_n with $n \geq 55$ and n ending in a 5, that is p_{55}, p_{65}, p_{75}, p_{85}, p_{95}, p_{105}, ...

Second statement is X, the statements for numbers ending in 1 could be, for instance, all true, or all false.

Third statement is X, for the same reason.

Fourth statement is F, since if it were true, then p_{52} would be true by induction, but we are given that it is false.