Ma2201/CS2022 | Quiz 0100

Discrete Mathematics

D Term, 2018

Print Name: ____________________

Sign: ____________________

1. (**6 points**) Let p and q be logical statements. Prove carefully by the double implication method that

$$(\neg p) \wedge (\neg q) = \neg(p \vee q)$$

♣ *Remember, a proof is a convincing argument. A list of equations is not an argument.*

First we prove $(\neg p) \wedge (\neg q) \Rightarrow \neg(p \vee q)$. *We assume that* $\neg p \wedge \neg q$ *is true, so both* $\neg p$ *and* $\neg q$ *and true, hence* p *and* q *are both false. Thus* $p \vee q$ *is false, hence* $\neg(p \vee q)$ *is true, as required. (Note, I ended this sentence with "as required", to match with the intention declared at the beginning of this paragraph to prove an implication. If I had left the first sentence off, which is perfectly legal, I could not end with "as required", but instead would have to explicitly state my conclusion, such as – "Therefore* $(\neg p) \wedge (\neg q) \Rightarrow \neg(p \vee q)$*." Remember, like with any writing, you have to consider the point of view of the reader. It is your job to make it clear to him, not his job to guess what you are doing and why. Don't write "as required" unless you have prepared the reader with the requirements to be met. I use that expression all the time in class, but it is not a catchphrase.)*

Next we prove $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$. *We may assume that* $\neg(p \vee q)$ *is true, so* $p \vee q$ *is false. So both* p *and* q *are false, that is,* $\neg p$ *and* $\neg q$ *are both true, so* $(\neg p) \wedge (\neg q)$ *is true, as required.*

Since $(\neg p) \wedge (\neg q) \Rightarrow \neg(p \vee q)$ *and* $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$ *are both true, we have*

$$(\neg p) \wedge (\neg q) = \neg(p \vee q).$$

♣

2. (**4 points**) Let p, q and r be statements. Suppose $p \Rightarrow (q \wedge r)$ is true. Label each of the following statements T if it *must* be true, F if it *must* be false, and X if the truth cannot be determined from the given information.

____ $q \wedge r$.

____ $q \vee r$.

____ $(p \wedge \neg q) \vee (p \wedge \neg r)$.

____ $(p \Rightarrow q) \vee (p \Rightarrow r)$.

For these we must only attend to the definition of implies.

The first and second are both X*, since if* p *is false, he have no information whatever about* q *and* r.

The third is F. *If* p *is true, then both* q *and* r *are true, so* $\neg q$ *and* $\neg r$ *are both false. So* $(p \wedge \neg q)$ *and* $(p \wedge \neg r)$ *are both false, so the OR statement is false.*

For the fourth one, if $p \Rightarrow (q \wedge r)$ *is true then* $p \Rightarrow q$ *and* $p \Rightarrow r$ *are both true, so* $(p \Rightarrow q \wedge (p \Rightarrow r)$ *is true, so the weaker OR statement is certainly true.*