Ma2201/CS2022 Discrete Mathematics D Term, MMXVI
Quiz 0111

Print Name: _____
Sign: _____

☐ Normal Quiz ☐ Replace 1 ☐ Double Down

1. (**3 points**) Find the multiplicative inverse of 13 in $\mathbb{Z}_{21}$.

♣ *Euclidean Algorithm:*

$$\begin{aligned} 5: \quad & 21 = 1 \cdot 13 + 8 \\ -3: \quad & 13 = 1 \cdot 8 + 5 \\ 2: \quad & 8 = 1 \cdot 5 + 3 \\ -1: \quad & 5 = 1 \cdot 3 + 2 \\ 1: \quad & 3 = 1 \cdot 2 + 1 \end{aligned}$$

Adding with the coefficients to the left gives

$$(5)(21) + (-8)(13) = 1$$

so $21 - 8 = 13$ *is the multiplicative inverse 13.*
Check $13^2 = 169 = 8 \cdot 21 + 1$.

2. (**4 points**) Let $A = \{\emptyset, \{\emptyset\}, 0\}$. Label each of the following TRUE or FALSE.

a) ____ $A \in \mathcal{P}(A)$. ♣ *TRUE: A is a subset of itself.*

b) ____ $\{\emptyset\} \subseteq \mathcal{P}(A)$. ♣ *TRUE:* $\emptyset$ *is an element of A, so* $\{\emptyset\} \subseteq A$,

c) ____ $|A \cap \mathcal{P}(A)| = 0$. ♣ *FALSE:* $A \cap \mathcal{P}(A) = \{\emptyset, \{\emptyset\}\}$

d) ____ $\mathcal{P}(\mathcal{P}(\emptyset)) \subseteq \mathcal{P}(A)$. ♣ *TRUE: Every set is a subset of itself.*

3. (**3 points**) Let p, q and r be statements and let $(p \Rightarrow q) \wedge (\neg q \Rightarrow r)$ be true.
Which of the following MUST be true.

a) ____ $\neg p \vee r$.

♣ $(p \Rightarrow q)$ is the same as $q \vee \neg p$, and $(\neg q \Rightarrow r)$ is the same as $r \vee q$, so if q is true, then $(p \Rightarrow q) \wedge (\neg q \Rightarrow r)$ true regardless of the values of p and r, so neither a) nor b) must be true.

b) ____ $p \Rightarrow \neg r$.

c) ____ $(p \Rightarrow r) \vee \neg(r \vee \neg p)$.

♣ $p \Rightarrow r$ is the same as $r \vee \neg p$ by definition, so either it or its negation must be true.