Ma2201/CS2022 # Discrete Mathematics D Term, MMXVI

Quiz 0101

Print Name: ____________________

Sign: ____________________

1. (**6 points**) Use the Euclidean Algorithm to find $\gcd(119, 85)$.
Find λ and μ so that $\lambda \cdot 119 + \mu \cdot 85 = \gcd(119, 85)$.

♣ *Euclidean Algorithm:*

$$\begin{aligned} 119 &= 1 \cdot 85 + 34 \\ 85 &= 2 \cdot 34 + 17 \\ 34 &= 2 \cdot 17 + 0 \end{aligned}$$

So $\gcd(119, 85) = 17$.

(Really? $85 = 5 \cdot 17$*,* $119 = 7 \cdot 17$*, ok – I believe it. That Euclidean Algorithm is always right.)*

We add

$$\begin{aligned} -2: &\qquad 119 = 1 \cdot 85 + 34 \\ 1: &\qquad 85 = 2 \cdot 34 + 17 \end{aligned}$$

and get

$$(-2)(119) + (3)(85) = 17$$

2. (**4 points**) Let p, and q be primes, $p \neq q$. Label each of the following true or false.

___ $2p + q$ must be prime.

♣ *Sounds like wishful thinking. Let's see ... if* $p = 3$ *and* $q = 2$ *then* $2p + q = 8$*, which is not prime. FALSE.*

___ $\gcd(p^2 + q^2, pq) = 1$.

♣ pq *only has divisors* 1*,* p*,* q *and* pq*. Since* $p \mid p^2$*, if* $p \mid (p^2 + q^2)$*, then* $p \mid q^2$ *which is impossible since* $p \neq q$*. And the same for* p*,so the greatest common divisor is* 1*. TRUE.*

___ $3p^2 - 5q^3$ cannot be zero.

♣ *If* $3p^2 = 5q^3$ *then* 3 *is in the prime factorization of* $5q^3$*, so* $q = 3$*. By the same token* $p = 5$*, so* $3p^2 = 5q^3$ *says* $27 = 125$*. TRUE.*

(Or you could say the prime factorization of $3p^2$ *has* 3 *primes, and the prime factorization of* $5q^3$ *has four, violating uniqueness.)*

___ For any integers n and m, $\gcd(n, m^2) = \gcd(n^2, m) \Rightarrow \gcd(n, m) = 1$.

♣ *Consider* $n = 6$ *and* $m = 10$*.* $\gcd(6, 100) = 2 = \gcd(10, 36) = 2$*, but* $\gcd(6, 10) = 2$ *also. FALSE.*

(What is true if $\gcd(n, m^2) = \gcd(n^2, m)$ *is that every prime occurring the both the prime factorization of* n *and* m *must have the same exponent in both.)*