Ma2201/CS2022 **Discrete Mathematics** D Term, MMXVI

Quiz 0100

Print Name: ____________________

Sign: ____________________

1. (**6 points**) Prove by induction that $n^2 + 5n + 1$ is odd for all $n \geq 0$.

♣ *Proof by induction.*

Base Case: $n = 0$. *The statement is that* $0^2 + 5(0) + 1 = 1$ *is odd, which is true.*

Inductive Step. We want to show that

$$(n^2 + 5n + 1) \text{ odd} \implies ((n+1)^2 + 5(n+1) + 1) \text{ odd.}$$

Let $n^2 + 5n + 1$. *(This is the inductive hypothesis.) Then* $((n+1)^2 + 5(n+1) + 1) = n^2 + 2n + 1 + 5n + 5 + 1 = (n^2 + 5n + 1) + 2(n+1)$. *We have that* $n^2 + 5n + 1$ *is odd by the inductive hypothesis, and* $2(n+1)$ *is even since it is divisible by 2, so* $((n+1)^2 + 5(n+1) + 1)$ *is odd, as required.*

Since the basis case and the inductive step are both true, the statement is true for all $n \geq 0$ *by induction.*

2. (**4 points**) Let p and q be statements. Show that $(p \vee q) \wedge (\neg p \vee r) \Rightarrow (q \vee r)$

♣ *There are many ways to correctly do this. The most tedious and least instructive is via truth tables.*

Best to show directly:
Let $(p \vee q) \wedge (\neg p \vee r)$, *so both* $p \vee q$ *and* $\neg p \vee r$ *are true.*
Either p *or* $\neg p$ *is true.*
  If p *is true, then* $\neg p$ *is false and* $\neg p \vee r$ *gives* r *is true, so* $q \vee r$ *is true.*
  If p *is false,* $p \vee q$ *gives* q *is true, so* $q \vee r$ *is true.*
So in either case, $q \vee r$ *is true.*

Another method is to use the distributive law first:

$$(p \vee q) \wedge (\neg p \vee r) = (p \vee \neg p) \wedge (p \vee r) \wedge (q \vee \neg p) \wedge (q \vee r).$$

If $(p \vee q) \wedge (\neg p \vee r)$ *is true, each of* $(p \vee \neg p)$, $(p \vee r)$, $(q \vee \neg p)$ *and* $(q \vee r)$ *is true, so* $(q \vee r)$ *is true.*