Ma2201/CS2022 **Discrete Mathematics** D Term, MMXVI

Quiz 0011

Print Name: ____________________

Sign: ____________________

1. (**6 points**) A password for the website `www.youcantrustus.ru` has 8 symbols from $A \cup D$, where $A = \{a, b, c, \ldots, x, y, z\}$ is the set of characters and $D = \{0, 1, 2, 3, \ldots, 8, 9\}$ is the set of digits. For the password to be acceptable, it must either start with three digits, or end with four characters, or consist only of symbols from $\{1, 2, 3, a, b, c\}$.

How many legal passwords are there?

♣ *We use inclusion exclusion. Let X = set of passwords with the first three symbols in D, Y be the set of passwords whose last four symbols are in A, and Z be the passwords with all symbols from $\{1, 2, 3, a, b, c\}$.*

The acceptable passwords must satisfy are least one of the conditions so we want to compute $|X \cup Y \cup Z|$ and so use inclusion exclusion so that each computation only uses the multiplicative principle.

First we compute: $|X| = 10^3 \cdot 36^5$, $|Y| = 36^4 \cdot 26^4$, and $|Z| = 6^8$.

Next we compute $|X \cap Y| = 10^3 \cdot 36 \cdot 26^4$, $|X \cap Z| = 3^3 \cap 6^5$, and $|Y \cap Z| = 6^4 \cap 3^4$, and

Lastly we compute $|X \cap Y \cap Z| = 3^3 \cdot 6 \cdot 3^4$.

Now, using inclusion exclusion we have

$$\begin{aligned} |X \cup Y \cup Z| &= |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z| \\ &= (10^3 \cdot 36^5) + (36^4 \cdot 26^4) + (6^8) \\ &\quad -(10^3 \cdot 36 \cdot 26^4) - (3^3 \cap 6^5) - (6^4 \cap 3^4) \\ &\quad +(3^3 \cdot 6 \cdot 3^4) \end{aligned}$$

2. (**4 points**) Label each of the following TRUE or FALSE.

a) ____ $|\mathcal{P}(\{1, 2, 3, 4, 5, 6\})| < |\mathcal{P}_4(\{1, 2, 3, 4, 5, 6, 7, 8\})|$

♣ *TRUE: $|\mathcal{P}(\{1, 2, 3, 4, 5, 6\})| = 2^6 = 64$. $|\mathcal{P}_4(\{1, 2, 3, 4, 5, 6, 7, 8\})| = \binom{8}{4} = 70$.*

b) ____ $|\mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$

♣ *TRUE: Finite product of countable sets is countable.*

c) ____ $|\mathbb{N} \times \mathbb{N}| = |\mathbb{Q}|$

♣ *TRUE: Both $\mathbb{N} \times \mathbb{N}$ and $\mathbb{Q}$ are countable.*

d) ____ $|\mathcal{P}(\mathbb{N})| > |\mathbb{Q} \times \mathbb{Q}|$

♣ *TRUE: $\mathcal{P}(\mathbb{N})$ is uncountable. $\mathbb{Q} \times \mathbb{Q}$ is countable.*