Ma2201/CS2022 Quiz 0010

Discrete Mathematics

D Term, MMXVI

Print Name: ____________________

Sign: ____________________

1. (**6 points**) Use the double inclusion method to prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

(If you need more room, please use the back.)

♣ *First show* $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$
Let $x \in A \cup (B \cap C)$. *So either* $x \in A$ *or* $x \in B \cap C$.
If $x \in A$, *then* $x \in A \cup B$ *and* $x \in A \cup C$, *so* $x \in (A \cup B) \cap (A \cup C)$, *as required.*
On the other hand, if $x \in B \cap C$, *then* $x \in B$ *and* $x \in C$. *Since* $x \in B$, $x \in A \cup B$. *Since* $x \in C$, $x \in A \cup C$, *so* $x \in (A \cup B) \cap (A \cup C)$, *as required.*
So in either case $x \in (A \cup B) \cap (A \cup C)$.

Second we show $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$
Let $y \in (A \cup B) \cap (A \cup C)$. *So* $x \in A \cup B$ *and* $x \in A \cup C$. *Either* $y \in A$ *or* $y \notin A$.
If $y \in A$, *then* $y \in A \cup (B \cap C)$ *as required.*
If $y \notin A$, *then since* $y \in A \cup B$, $y \in B$. *Since* $y \in A \cup C$, $y \in C$. *So* $y \in B \cap C$, *hence* $y \in A \cup (B \cap C)$.
So in either case $y \in A \cup (B \cap C)$.

2. (**4 points**) Let $A = \{\emptyset, \{\emptyset\}, 0\}$. Label each of the following TRUE or FALSE.
♣ *A has* 3 *elements so* $\mathcal{P}(A)$ *has* $2^3 = 8$ *elements:*

$$\{\ \emptyset,\ \{\emptyset\},\ \{\{\emptyset\}\},\ \{0\},\ \{\emptyset, \{\emptyset\}\},\ \{\emptyset, 0\},\ \{\{\emptyset\}, 0\},\ \{\emptyset, \{\emptyset\}, 0\}\ \}$$

a) ____ $A \subseteq \mathcal{P}(A)$. ♣ *FALSE:* $0 \in A$, *but* $0 \notin \mathcal{P}(A)$ *since all elements of* $\mathcal{P}(A)$ *are sets. It is true that* $A \in \mathcal{P}(A)$.

b) ____ $\emptyset \subseteq \mathcal{P}(A)$. ♣ *TRUE:* $\emptyset$ *is a subset of every set. (In this case, it is also true that* $\emptyset \in \mathcal{P}(A)$.*)*

c) ____ $A \cap \mathcal{P}(A) = \emptyset$. ♣ *FALSE:* $A \cap \mathcal{P}(A) = \{\emptyset, \{\emptyset\}\}$

d) ____ $\mathcal{P}(A) \subseteq \mathcal{P}(A)$. ♣ *TRUE: Every set is a subset of itself.*