Exercises from Lectures 1 and 2

Discreteness

1. Children at a birthday party take turns striking a piñata filled with candy with a stick. What aspects of this situation are discrete? What aspects are continuous. To what extent could discrete mathematics be involved.

2. Consider the game of baseball. What aspects of the game are continuous? What aspects are discrete? Give an example of a discrete mathematical problem arising in baseball.

3. Suppose you wish to communicate a message to a friend. Give an example of a discrete method. Give an example of a continuous method.

4. Order the following musical instruments with respect to discreteness: guitar, violin, piano, tuba, bugle, trombone, bass drum and clarinet. Justify your choice.

5. Which of the following statements in chemistry are discrete?

   - The Ideal Gas Law: $PV = nRT$,
   - Dalton's Law: Equal volumes of gas contain the same number of molecules,
   - Chemical masses react in small whole number ratios,
   - Given a 1 molar solution with a ph of 5 ...
   - The alkali earth metals have an electronegativity of +1,
   - Metals are ductile and have high conductivity,
   - Water expands as it freezes.

6. Which of the following physical laws is discrete?

   - Every action has an equal an opposite reaction,
   - In the absence of applied torques, angular momentum is conserved,
   - Force equals mass times acceleration,
   - The half-life of carbon 14 is 5700 years.

7. Which of the following concepts from music are discrete?

   - The diatonic scale,
   - forte,
   - octave,
   - she sings flat,
   - she's singing an A at 404Hz,
   - semi-quaver,

- ritardando.

8. Which of the following controls of a typical automobile are discrete?
   - shifting (automatic transmission),
   - shifting (standard transmission),
   - the brake,
   - the clutch,
   - the steering wheel,
   - cruise control,
   - windshield wipers,
   - the parking brake.

Multiplicative Principle

1. A Fifth grader has 5 quarters, 3 dimes, a nickel and 10 pennies. How many different kinds of stacks of coins can he make such that the coins in the stack decrease in radius from the bottom to the top.

2. A piano has 88 keys. A composer defines an 8 note melody to consist of a sequence of 8 notes which ends on middle C such that no two successive notes are more than 5 keys apart. Can we use the multiplicative principle to count the number of 8 note melodies? What about the 18 note melodies?

3. In a tic-tac-toe game, each of the nine squares can either be blank, or have an X or an O, so in playing tic-tac-toe there are 3^9 states.

   If X wins after 9 turns, there will be 9 ways to place the First X, 8 ways to place the next O and so on, giving $9 \cdot 8 \cdot \cdots \cdot 1 = 9!$different 9 move tic-tac-toe games in which X wins.

   Which of these computations is done correctly with the multiplicative principle? How is independence satisfied or violated?

4. A restaurant serves a luncheon special with a choice of two appetizers, three choices of entrée, 4 choices of salad, and 5 choices of desert. How many different meals are possible?

5. Two roommates live an a dorm. Their mother's both shop at the same stores and have the same taste, so they both have the same 3 pairs of shoes, the same 3 pairs of pants, the same 3 shirts, and the same three sweaters. How many ways can they dress so that are not both dressed exactly the same.

6. A password for the website `www.sendmespam.com` must have exactly 8 letters, with upper and lower case letter indistinguishable. If you try to hack an account by trying each possible password, and the website allows you to try one password per millisecond, how long will it take, in the worst case, to break into the account?

   If the upper and lower case letter are distinguished, will it take twice as long?

7. In response to complaints, www.sendmespam.com wants to make its passwords stronger, but doesn't want to anger its customers. The suggestions to either allow passwords to be at most 8 letters long, or to change to passwords exactly 9 characters long. Which would yield more passwords.

8. Mr. Snipp's favorite knight move in chess was from King's Bishop 3 to King 5.

   Compute how may ways there are to move a knight on a standard 8×8 chessboard.

   [Hint: The multiplicative principe cannot be used naively.]

9. There is a survey with 20 questions. The first question is whether you are male or female. The second question is multiple choice with four options. The third question is multiple choice with 8 options, and for each subsequent question the number of choices is double that of the previous question. How many different survey responses can there be?

   ♣ *The most that there can be is when the responses are independent, and we can use the multiplicative principle. The number of responses doubles with each questions, and for the i'th question there are* 2^i *responses. So for the twenty questions you get*

   $$2^1 \cdot 2^2 \cdot 2^3 \cdot \cdots \cdot 2^2 0 = 2^{1+2+3+\cdots 20}$$

   which algebraic expression is fine for the answer. Adding up $\sum_{k=1}^{20} k$ *is not required, but many of you know from calculus it is*

   $$\sum_{k=1}^{20} k = (20)(20+1)/2 = 210,$$

   so

   $$2^{210}$$

   is correct also. ♣

10. There is a 3×2 square grid, and on each square you can place a stack of three US coins, (pennies, nickels, dimes, quarters, half-dollars, and dollars).

    We want to compute how many ways are there to do this?

You can solve this problem in two ways depending on whether turning the grid matters or not, so if you use all quarters except for one stack of three pennies is one corner, is that a single solution, our is it four solutions distinguishing the cases where the pennies are top-left, bottom-left, top-right, or bottom-right.

Pick one of the interpretations, solve that, and then say whether the other questions has an answer more or less that what you computed.

11. There is a large bar of Swiss chocolate laid out in an array of 6×8 squares. Typically, the bars are shared by breaking them along the ridges.

    If you break the bar initially on a horizontal ridge, the break is of length 6, and if you break on an initial vertical ridge, the break of of length 8.

    For example, you could start with a vertical break on the second ridge and get a 2×8 and a 4×8 piece, than then break the 4×8 piece on the 4th horizontal ridge and get two 2×2, and two 4×4 pieces.

    You cannot stack the pieces, you can only break one piece at a time.

    If you want to end up with the bar completely broken up, what is the method of breaking which has the fewest breaks.