Linear Algebra

Print Name: ____________________
Sign: ____________________

Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 4 & 2 & 4 & 4 \\ 3 & 1 & 3 & 3 \end{bmatrix}$

1. (**3 pts**) Find a basis for the column space of A.

....

Row operations for not change the column dependencies, we can take the pivot columns of A.

A quickly row reduces to $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ so A has pivot columns 1 and 2, so a basis for the column space of A is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

2. (**2 pts**) Find a basis for the null space of A.

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The null space is the solutions of $A\mathbf{x} = \mathbf{0}$, so from the row reduced form we have free variables x_3 and x_4, and we can have a basis by choosing $x_3 = 1$, $x_4 = 0$ for one, and $x_3 = 0$, $x_4 = 1$ for the other:

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

3. (**2 pts**) Find a basis for the row space of A.

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Row operations for not change the row space, so we can take the pivot *rows* of the row echelon form:

$$\{[1\ 0\ 1\ 1], [0\ 1\ 0\ 0]\}$$

[Note: *For this particular example the pivot rows of the original also work. Why?*]

4. (**3 pts**) Find a basis for the null space of A^T.

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$A^T = \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 4 & 3 \\ 1 & 1 & 4 & 3 \end{bmatrix}$ which row reduces to $A^T = \begin{bmatrix} 1 & 1 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ with free variables x_3 and x_4, so with the same choice as before we get $\left\{ \begin{bmatrix} -4 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$