Ma2071 # Linear Algebra C Term, 2012

Quiz 4

Print Name: ____________________

Sign: ____________________

1. (**4 pts**) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & \pi \\ \pi^{-1} & 1 \end{bmatrix}$

   Circle each of the following which is true.

   $\boxed{\text{a)}}$ A is invertible $\quad \boxed{\text{b)}}$ $(AB)^{-1} = \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

   c) C is invertible $\quad$ d) CBA is non-singular

. . . .

$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ $\quad B^{-1} = \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix}$ $\quad (AB)^{-1} = B^{-1}A^{-1}$.

C^{-1} is undefined since $1 \cdot 1 - \pi \cdot \pi^{-1} = 0$.

For the last, you don't have to multiply out if you note that the columns of CBA are a linear combination of the columns of C, which in turn are dependent, so CBA is not invertible, in other words, singular.

2. (**3 pts**) Let $D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 4 & 4 & 4 & 0 \\ 8 & 8 & 8 & 8 \end{bmatrix}$.

   Compute D^{-1}.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 & 0 & 1 & 0 \\ 8 & 8 & 8 & 8 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 4 & 4 & 0 & -4 & 0 & 1 & 0 \\ 0 & 8 & 8 & 8 & -8 & 0 & 0 & 1 \end{array}\right] \to$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 8 & 8 & 0 & -4 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & -2 & 1 \end{array}\right] \to$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/2 & 1/4 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1/4 & 1/8 \end{array}\right].$$

So $D^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1/2 & 0 & 0 \\ 0 & -1/2 & 1/4 & 0 \\ 0 & 0 & -1/4 & 1/8 \end{bmatrix}$.

3. (**3 pts**) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

   Compute $A^2 + 2(A^{-1} + A)^T = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} + 2\left(\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)^T =$

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & -1 \end{bmatrix}$.