Ma2071 Quiz 3

Linear Algebra

C Term, 2012

Print Name: ____________________

Sign: ____________________

All problems refer to the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.

For each of the following statements mark it with T if it is true, F if it is false.

They are worth 1 point each.

1. __T__ The first three columns of A are linearly dependent.

   [Any set of vectors containing the zero vector is dependent.]

2. __T__ The last three columns of A span a plane through the origin.

   [The middle vector doesn't contribute anything, and the 4'th and 5'th columns are obviously linearly independent, so they span a plane through the origin.]

3. __F__ A has a pivot in position $(5, 1)$.

   [A is already in reduced row echelon form, and there are a leading 1's in positions $(1, 1)$, $(2, 2)$ and $(3, 4)$. There is no entry at all in the $(5, 1)$ position.]

4. __F__ A is the matrix of a linear transformation $F(\mathbf{x}) = A\mathbf{x}$ with domain $\mathbb{R}^3$.

   [The domain of F is $\mathbb{R}^5$.]

5. __F__ The first column of A is linearly dependent on the 2'nd and 3'rd columns.

   [Any linear combination of the second and third columns has a zero in the top spot, so cannot equal the first column]

6. __F__ The first column of A is linearly dependent on the 4'th and 5'th columns.

   [Same explaination as for 5]

7. __F__ The transformation $F(\mathbf{x}) = A\mathbf{x}$ is one to one.

   [Since the columns are dependent, the transformation is not one-to-one. Or, $F(\mathbf{0}) = \mathbf{0}$ and $F(\mathbf{e}_3) = \mathbf{0}$.]

8. ____ The transformation $G(\mathbf{x}) = A\mathbf{x} + 2\mathbf{x}$ is linear.

   [This problem had an error in the statement, can you find it?]

9. __T__ There is a unique fixed vector $\mathbf{a}$ such transformation $H(\mathbf{x}) = A\mathbf{x} + \mathbf{a}$ is linear.

   [A linear transformation must satisfy $H(\mathbf{0}) = \mathbf{0}$, so $\mathbf{a} = \mathbf{0}$ is the unique such vector.]

10. __T__ The middle column of A is dependent on the other four columns.

    [The zero vector is a linear combination of any set of vectors.]