Ma2071 | **Linear Algebra** | C Term, 2012

Quiz 2

Print Name: ____________________

Sign: ____________________

All five problems refer to the matrix $A = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ 2 & 8 & -2 & 1 \end{bmatrix}$.

For each problem, circle the correct answer.

1. For the matrix equation $A\mathbf{x} = \begin{bmatrix} 2 \\ -5 \\ 7 \\ 1 \end{bmatrix}$.

   a) the solutions form a line through the origin in $\mathbb{R}^4$. b) there is only the trivial solution.
   c) the solutions form a plane through the origin in $\mathbb{R}^4$. $\boxed{\text{d)}}$ None of the above.

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The equation refers to $\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ 2 & 8 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 7 \\ 1 \end{bmatrix}$

The matrix $A$ has one column with no pivot position, so there is one free variable and the solution set is a line, but the zero vector is not a solution, so the line does not pass through the origin.

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2. The matrix $A$

   $\boxed{\text{a)}}$ has a pivot in position $(3, 4)$. b) has a pivot in position $(4, 4)$.
   c) has a pivot in position $(4, 3)$. d) none of the above.

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The reduced row echelon form for $A$ has a pivot in the third row, fourth column.

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3. If $A$ is the augmented matrix of a linear system, that linear system has

   a) a unique solution. b) exactly two solutions, one of which is trivial.
   $\boxed{\text{c)}}$ no solutions. d) infinitely many solutions.

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The pivot at $(3, 4)$, in the augmenting column, indicates that there is no solution to the system

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & 8 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 7 \\ 1 \end{bmatrix}$$

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4. The columns of $A$ span

a) the set of solutions of the system $A\mathbf{x} = \mathbf{0}$. b) a line through the origin in $\mathbb{R}^4$.

c) a sphere of radius 1 centered at $\mathbf{0}$. $\boxed{\text{d)}}$ none of the above.

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a) is incorrect, just try the first column.

b) is incorrect, otherwise all the columns of $A$ would be scalar multiples of one another.

c) is incorrect since the span of a collection of vectors in $\mathbb{R}^4$ is the origin, a line through the origin, a plane through the origin, a three dimensional space through the origin, or all of $\mathbb{R}^4$.

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5. If the vector $\mathbf{z}$ is a linear combination of the columns of $A$ then

a) $\mathbf{z}$ must be a solution of the system $A\mathbf{x} = \mathbf{0}$. b) $\mathbf{z}$ cannot be the zero vector.

$\boxed{\text{c)}}$ $\mathbf{z}$ must lie in the span of the columns of $A$. d) none of the above.

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The span of a set of columns is by definition the set of all linear combinations of then.

Note that the zero vector is always a linear combination of a set of vectors by taking all the scalars to be zero.

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