Ma2071
Quiz 1

Discrete Mathematics

C Term, 2012

Print Name: _____

Sign: _____

1. The matrix $A = \begin{bmatrix} 1 & 1 & 2 & 2 & 7 \\ 1 & 1 & 2 & 3 & 8 \\ 0 & 1 & 1 & 2 & 4 \end{bmatrix}$ is the *augmented* matrix of a linear system.

a) (**2 pts**) Give the system of linear equations.

$$\begin{array}{rcrcrcrcl} x & + & y & + & 2z & + & 2w & = & 7 \\ x & + & y & + & 2z & + & 3w & = & 8 \\ & + & y & + & z & + & 2w & = & 4 \end{array}$$

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b) (**2 pts**) Transform A into row echelon form, showing clearly the steps you follow.

$$\begin{bmatrix} 1 & 1 & 2 & 2 & 7 \\ 1 & 1 & 2 & 3 & 8 \\ 0 & 1 & 1 & 2 & 4 \end{bmatrix} \xrightarrow{R2 \leftarrow R2 - R1} \begin{bmatrix} 1 & 1 & 2 & 2 & 7 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 & 4 \end{bmatrix} \xrightarrow{R2 \leftrightarrow R3} \begin{bmatrix} 1 & 1 & 2 & 2 & 7 \\ 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

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c) (**2 pts**) Transform A into reduced row echelon form, showing clearly the steps you follow. (You many continue from where you left of in part b)

$$\begin{bmatrix} 1 & 1 & 2 & 2 & 7 \\ 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1 \leftarrow R1 - R2} \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R2 \leftarrow R2 - 2R3} \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

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d) (**2 pts**) If possible, give two distinct solutions of the system. How many solutions are there?

z is the free variable. Taking $z = 0$ gives the solution $(3, 2, 0, 1)$.
Taking $z = 1$ gives the solution $(2, 1, 1, 1)$.
There are infinitely many solutions.

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e) (**2 pts**) What are the pivot positions of the matrix

The pivot positions are the *positions* of the leading 1's in the reduced row echelon form. For the matrix A those positions are $(1, 1)$, $(2, 2)$, and $(3, 4)$.