Ma1023
Quiz 6 B

Calculus III

A Term, 2013

Print Name: ____________________

O Normal O Double Down O Triple Down The Double-Down Quiz

1. (3 pts) Consider the function $r(\theta) = \cos(2\theta)$.

Label each of the following with **T** if it must be true, **F** if it must be false, and **X** if it cannot be determined from the given information.

__T__ The graph of $r(\theta)$ has a tangent of slope 1 at $\pi/4$ in the xy-plane

$\cos(2\pi/4) = 0$, so the polar graph is tangent at the origin to the ray at $\pi/4$, which has slope 1.

__F__ The graph of $r(\theta)$ has a horizontal tangent at $\pi/4$ in the xy-plane

See above.

__F__ The graph of $r(\theta)$ has a horizontal tangent at $\pi/4$ in the θr-plane

The horizontal tangents in the θr-plane correspond to the maxes and mins of $r = \cos(2\theta)$.

2. (3 pts) Compute carefully and neatly the following integral, showing all steps required.

$$\int_3^\infty (1+x)e^{-4x}\,dx = \lim_{b\to\infty} [(-5-4b)e^{-4b}/16] - [(-5-4\cdot 3)e^{-4\cdot 3}/16]$$

$$= -[(-5-12)e^{-4\cdot 3}/16] = (17/16)e^{-12}$$

Sidebar: $\int (1+x)e^{-4x}\,dx \overset{\substack{u=1+x \\ du=dx}}{} \overset{\substack{dv=e^{-4x} \\ v=(-1/4)e^{-4x}}}{=} (1+x)(-1/4)e^{-4x} - \int (-1/4)e^{-4x}dx$

$= (1+x)(-1/4)e^{-4x} - (-1/4)(-1/4)e^{-4x} = (-5-4x)e^{-4x}/16$

Sidebar: $\lim_{b\to\infty} (-5-4b)e^{-4b} = \lim_{b\to\infty} \frac{-5-4b}{e^{4b}} \overset{\text{l'h rule}}{=} \lim_{b\to\infty} \frac{-4}{4e^{4b}} = 0$

3. (4 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{x/3}$$

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{x/3} = \lim_{x\to\infty} e^{(x/3)\ln(1+\frac{1}{x})} = e^{1/3} = \sqrt[3]{e}.$$

Sidebar: $\lim_{x\to\infty} (x/3)\ln(1+\frac{1}{x}) = (1/3)\lim_{x\to\infty} \frac{\ln(1+\frac{1}{x})}{1/x} \overset{\text{l'h rule}}{=} (1/3)\lim_{x\to\infty} \frac{(1+\frac{1}{x})^{-1}(-1/x^2)}{-1/x^2}$

$= (1/3)\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{-1} = (1/3)\cdot 1 = 1/3.$