Ma1023 Quiz 6 A

Calculus III

A Term, 2013

Print Name: ____________________

○ Normal ○ Double Down ○ Triple Down The Double-Down Quiz

1. (3 pts) Consider the function $r(\theta) = \cos(4\theta)$.

Label each of the following with **T** if it must be true, **F** if it must be false, and **X** if it cannot be determined from the given information.

__T__ The graph of $r(\theta)$ has a horizontal tangent at $\pi/4$ in the θr-plane
$\cos(4\theta)$ has a local min at $\pi/4$.

__F__ The graph of $r(\theta)$ has a horizontal tangent at $\pi/4$ in the xy-plane
At the local min the polar graph of $r = \cos(4\theta)$ will be perpendicular to the ray $\pi/4$.

__T__ The graph of $r(\theta)$ has a tangent of slope $\tan(\pi/8)$ at $\pi/8$ in the xy-plane
$\cos(4\pi/8) = 0$, so the polar graph will pass through the origin tangent to the ray $\pi/8$.

2. (3 pts) Compute carefully and neatly the following integral, showing all steps required.

$$\int_4^\infty (1+x)e^{-3x}\,dx = \lim_{b\to\infty}[(-4-3b)e^{-3b}/9] - [(-4-3\cdot 4)e^{-3\cdot 4}/9]$$
$$= -[(-4-12)e^{-3\cdot 4}/9] = 16/9e^{-12}$$

Sidebar: $\displaystyle\int (1+x)e^{-3x}\,dx \overset{\substack{u=1+x \\ du=dx}}{} \overset{\substack{dv=e^{-3x} \\ v=(-1/3)e^{-3x}}}{=} (1+x)(-1/3)e^{-3x} - \int (-1/3)e^{-3x}dx$
$= (1+x)(-1/3)e^{-3x} - (-1/3)(-1/3)e^{-3x} = (-4-3x)e^{-3x}/9$

Sidebar: $\displaystyle\lim_{b\to\infty}(-4-3b)e^{-3b} = \lim_{b\to\infty}\frac{-4-3b}{e^{3b}} \overset{\text{l'h rule}}{=} \lim_{b\to\infty}\frac{-3}{3e^{3b}} = 0$

3. (4 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x/2} = \lim_{x\to\infty} e^{(x/2)\ln(1+\frac{1}{x})} = e^{1/2} = \sqrt{e}.$$

Sidebar: $\displaystyle\lim_{x\to\infty}(x/2)\ln(1+\frac{1}{x}) = (1/2)\lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{1/x} \overset{\text{l'h rule}}{=} (1/2)\lim_{x\to\infty}\frac{(1+\frac{1}{x})^{-1}(-1/x^2)}{-1/x^2}$
$\displaystyle = (1/2)\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{-1} = (1/2)\cdot 1 = 1/2.$