Ma1023 # Calculus III A Term, 2013

Quiz 4 B Print Name: ____________________

1. (2 pts) Let $\{a_k\}_{k=0}^{\infty}$ be an infinite sequence of positive terms and let $s_n = \sum_{k=0}^{n} a_k$ be its sequence of partial sums. Label the following as **T** for TRUE or **F** for FALSE or **X** if it cannot be determined from the given information.

<u>X</u> The sequence $\{a_k\}$ is monotonically increasing.

[But we can conclude s_n is monotonically increasing.

<u>T</u> $\lim_{n\to\infty} s_n = \sum_{n=0}^{\infty} a_n$

[Definition of the sum of a series – the limit of the partial sums]

2. (4 pts) Compute the limit of the sequence $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n$

$$\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n \overset{\text{by interpolation}}{=} \underbrace{\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x}_{\text{of the form } 1^\infty} = \lim_{x\to\infty} e^{x\ln(1+\frac{3}{x})} = e^3$$

The limit of the sequence is e^3;

$$\lim_{x\to\infty} x\cdot\ln\left(1+\frac{3}{x}\right) = \underbrace{\lim_{x\to\infty}\frac{\ln(1+\frac{3}{x})}{1/x}}_{\text{of the form } \frac{0}{0}} \overset{\text{by l'Hopital's rule}}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\frac{3}{x}}\cdot(-3/x^2)}{-1/x^2} = \lim_{x\to\infty}\frac{-3}{\frac{1+3/x}{-1}} = \lim_{x\to\infty}\frac{3}{1+3/x} = 3/1 = 3$$

Thanks to Shawn Wile

3. (3 pts) Determine whether or not the following infinite series is geometric, determine whether it converges, and if so, find its sum.

$$\sum_{n=3}^{\infty}\frac{(-1)^{k+1}2^{3k+1}}{3^{2n-1}}$$

Ratio $\frac{a_{k+1}}{a_k} = \frac{\left(\frac{(-1)^{k+2}\,2^{3(k+1)+1}}{3^{2(k+1)-1}}\right)}{\left(\frac{(-1)^{k+1}\,2^{3k+1}}{2^{2k-1}}\right)} = \frac{(-1)(2^3)}{3^2} = -\frac{8}{9}$ Does not depend on k. The series is geometric.

$|r| = \left|-\frac{8}{9}\right| = \frac{8}{9} < 1$

The series converges to $a\left(\frac{1}{1-r}\right)$

$a = \frac{(-1)^4\,2^{9+1}}{3^{6-1}} = \frac{2^{10}}{3^5}$

$a\left(\frac{1}{1-r}\right) = \frac{2^{10}}{3^5}\left(\frac{1}{1-(-\frac{8}{9})}\right) = \frac{2^{10}}{3^5}\left(\frac{1}{\frac{17}{9}}\right) = \frac{2^{10}}{3^3\cdot 17}$

Thanks to Yutong Li