Ma1023 **Calculus III** A Term, 2013

Quiz 4 A Print Name: ___________________

1. (2 pts) Let $\{a_k\}_{k=0}^{\infty}$ be an infinite sequence of positive terms and let $s_n = \sum_{k=0}^{n} a_k$ be its sequence of partial sums. Label the following as **T** for TRUE or **F** for FALSE or **X** if it cannot be determined from the given information.

T The sequence $\{s_n\}$ is monotonically increasing.

F $\lim_{n\to\infty} a_n = \sum_{n=0}^{\infty} s_n$

[The sum of the series is the limit of the partial sums, not the reverse.

2. (4 pts) Compute the limit of the sequence $\lim_{n\to\infty}\left(1-\frac{2}{n}\right)^n$

consider $f(x) = (1-\frac{2}{x})^x$

$\lim_{x\to\infty} (1-\frac{2}{x})^x$ of the form 1^∞

$\lim_{x\to\infty} e^{x\ln(1-\frac{2}{x})} = \lim_{x\to\infty} e^{\frac{\ln(\frac{x-2}{x})}{\frac{1}{x}}}$ by L'H rule $= \lim_{x\to\infty} e^{\frac{\frac{x}{x-2}(\frac{2}{x^2})}{-\frac{1}{x^2}}}$ by L'H rule $= \lim_{x\to\infty} e^{\frac{x}{(x-2)}(\frac{2}{x^2})(-\frac{x^2}{1})} = \lim_{x\to\infty} e^{\frac{-2x}{x-2}} =$ by L'H rule

$\lim_{x\to\infty} e^{\frac{-2}{1}} = e^{-2} = \frac{1}{e^2}$

therefore, since $f(n) = (1-\frac{2}{n})^n$, the limit of the sequence $(1-\frac{2}{n})^n$ is $\boxed{e^{-2}} = \boxed{\frac{1}{e^2}}$

Thanks to Alexandra D'Ordine

3. (3 pts) Determine whether or not the following infinite series is geometric, determine whether it converges, and if so, find its sum.

$$\sum_{n=5}^{\infty} \frac{7^{k+4}\ln(2)}{2^{3+3k}}$$

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$$\frac{a_{k+1}}{a_k} = \frac{\left(\frac{7^{(k+1)+4}\ln(2)}{2^{3+3(k+1)}}\right)}{\left(\frac{7^{k+4}\ln(2)}{2^{3+3k}}\right)} = \frac{(7^{k+5}\ln(2))(2^{3+3k})}{(2^{6+3k})(7^{k+4}\ln(2))} = \frac{7\ln(2)}{2^3\ln(2)} = \frac{7}{2^3}$$

constant

So the series is geometric. $\boxed{|r| = \frac{7}{8} < 1 \text{ so the series converges}}$

$\sum_{k=5} ar^n = \frac{a}{1-r}$ $r = \frac{7}{8}$ $a = \frac{7^9\ln(2)}{2^{18}}$ so, $\frac{\left(\frac{7^9\ln(2)}{2^{18}}\right)}{1-\frac{7}{8}} = \frac{\left(\frac{7^9\ln(2)}{2^{18}}\right)}{1/8} =$

$$\boxed{\frac{(8)(7^9\ln(2))}{2^{18}}}$$

1 of 1

Thanks to Alison Marx