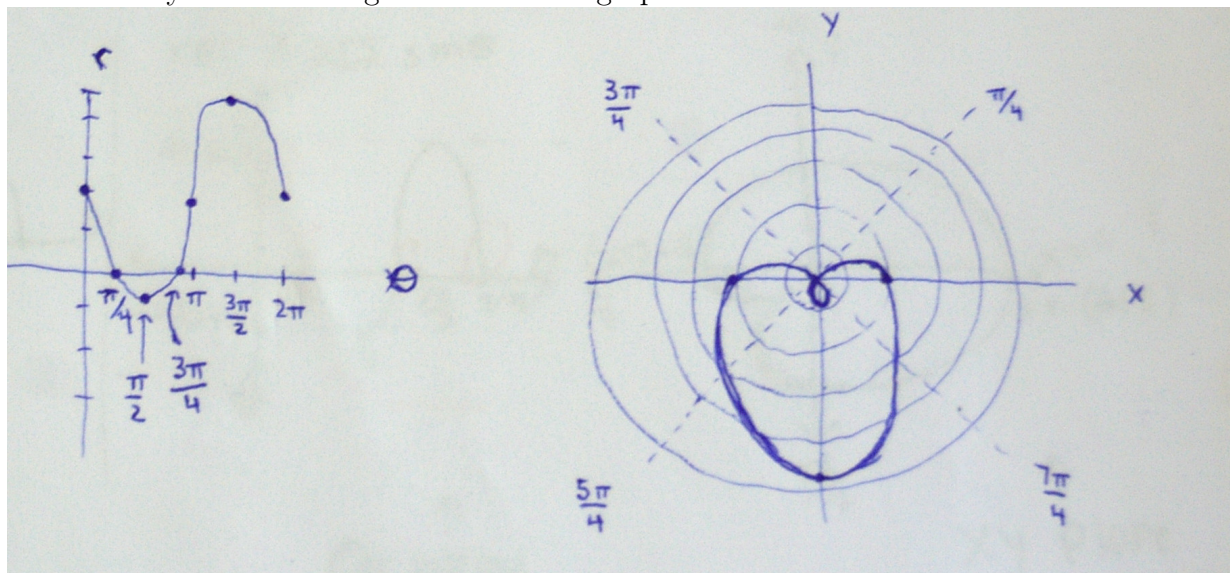# Calculus III

Print Name: _____

1. (5 pts) Sketch the graph of $r(\theta) = 2 - 2\sqrt{2}\sin(\theta)$ in the θr plane.
   Sketch as well the graph of $r(\theta) = 2 - 2\sqrt{2}\sin(\theta)$ in the xy plane.
   Label clearly all interesting features of the graphs.

[Thanks Joseph Heng]

2. (2 pts) What is the slope of the tangent line to the curve in the xy plane at $\theta = \pi/2$.
   [Note dy/dx is the slope in the xy plane. $dr/d\theta$ is the slope in the θ r plane.]

3. (3 pts) Write down, but do not compute, a definite integral which gives the area enclosed
   by the polar graph in problem 1. (No area counted twice.)

$$X = r\cos\theta = (2 - 2\sqrt{2}\sin\theta)\cos\theta$$

$$Y = r\sin\theta = (2 - 2\sqrt{2}\sin\theta)\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}\, 2\sin\theta - 2\sqrt{2}\sin^2\theta}{\frac{d}{d\theta}\, 2\cos\theta - 2\sqrt{2}\sin\theta\cos\theta} = \frac{2\cos\theta - 4\sqrt{2}\sin\theta\cos\theta}{2\sin\theta - 2\sqrt{2}\cos^2\theta + 2\sqrt{2}\sin^2\theta}$$

$$\frac{dy}{dx}\Big|_{\theta = \pi/2} = \frac{0 - 6}{2 - 0 + 2\sqrt{2}} = 0 \quad \boxed{\text{Slope at } \theta = \frac{\pi}{2} \text{ is } 0}$$

3. (3 pts) Write down, but do not compute, a definite integral which gives the area enclosed
by the polar graph in problem 1. (No area counted twice.)

$$\boxed{A = \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}\left(2 - 2\sqrt{2}\sin\theta\right)^2 d\theta}$$

[Thanks Peter Melender]