Ma1023 Test 1

Calculus III

A Term, 2013

Name: ____________________

1. (3 pts) Suppose $f(x)$ is a function for all x and that
$\int_0^\infty f(x)\,dx = \infty$.
Label each of the following with **T** if it must be true, **F** if it must be false, and **X** if it cannot be determined from the given information.

____ $\int_\pi^\infty f(x)\,dx = \infty$

T: $\int_0^\infty f(x)\,dx = \int_0^\pi f(x)\,dx + \int_\pi^\infty f(x)\,dx$ and since $f(x)$ is continuous, $\int_0^\pi f(x)\,dx$ is not improper, it is just a number.

____ $\int_0^\infty f(3x)\,dx = \infty$

T: Even without substituting, you are integrating the same heights as before, just with the widths shrunk by 1/3.

____ $\int_0^\infty \frac{1}{f(x)}\,dx = 0$

X: There is no way to tell. The reciprocal shrinks large numbers and expands small ones. You would have to recompute.

2. (3 pts) Compute carefully and neatly the following integral, showing all steps required.

$$\int_3^\infty \frac{1}{\sqrt{(1+5x)^3}}\,dx = \lim_{L\to\infty} \frac{-2}{5\sqrt{1+5L}} - \frac{-2}{5\sqrt{1+15}} = 0 + \frac{2}{20} = \frac{1}{10}$$

Antiderivative: $\int \frac{1}{\sqrt{(1+5x)^3}}\,dx \overset{\substack{u=1+5x\\ du=5dx}}{=} (1/5)\int u^{-3/2}\,du$

$= (1/5)(\frac{u^{-1/2}}{-1/2}) = (-2/5\sqrt{u}) = (-2/5\sqrt{1+5x})$

3. (4 pts) Compute carefully and neatly the following integral, showing all steps required.

$$\int_5^\infty xe^{-2x}\,dx = \lim_{L\to\infty}[-Le^{-2L}/2 - e^{-2L}/4] - [-5e^{-10}/2 - e^{-10}/4]$$

$= [(1/2)\lim_{L\to\infty}\frac{L}{e^{2L}}] - 0 + [11e^{-10}/4]$

$\overset{\text{by l'H}}{=} [(1/2)\lim_{L\to\infty}\frac{1}{2e^{2L}}] - 0 + [11e^{-10}/4] = 0 - 0 + [11e^{-10}/4] = 11e^{-10}/4$

Antiderivatives: $\int xe^{-2x}\,dx \overset{\substack{u=x \quad dv=e^{-2x}dx\\ du=dx \quad v=e^{-2x}/-2}}{=} -xe^{-2x}/2 - \int (e^{-2x}/-2)\cdot 1\,dx$

$= -xe^{-2x}/2 + (1/2)\int e^{-2x}\,dx = -xe^{-2x}/2 + (1/2)(e^{-2x}/-2)$

$= -xe^{-2x}/2 - e^{-2x}/4$

$\int e^{-2x}\,dx \overset{\substack{u=-2x\\ du=-2dx}}{=} (-1/2)\int e^u\,du = (-1/2)e^u = (-1/2)e^{-2x}$